IN THE SUPREME COURT OF WISCONSIN

No. 2023AP1399

REBECCA CLARKE, RUBEN ANTHONY, TERRY DAWSON, DANA GLASSTEIN, ANN GROVES-LLOYD, CARL HUJET, JERRY IVERSON, TIA JOHNSON, ANGIE KIRST, SELIKA LAWTON, FABIAN MALDONADO, ANNEMARIE MCCLELLAN, JAMES MCNETT, BRITTANY MURIELLO, ELA JOOSTEN (PARI) SCHILS, NATHANIEL SLACK, MARY SMITH-JOHNSON, DENISE (DEE) SWEET, AND GABRIELLE YOUNG,

Petitioners,

GOVERNOR TONY EVERS, IN HIS OFFICIAL CAPACITY; NATHAN ATKINSON, STEPHEN JOSEPH WRIGHT, GARY KRENZ, SARAH J. HAMILTON, JEAN-LUC THIFFEAULT, SOMESH JHA, JOANNE KANE, AND LEAH DUDLEY,

Intervenors-Petitioners

v.

WISCONSIN ELECTIONS COMMISSION; DON MILLIS, ROBERT F. SPINDELL, JR., MARK L. THOMSEN, ANN S. JACOBS, MARGE BOSTELMANN, AND CARRIE RIEPL, IN THEIR OFFICIAL CAPACITIES AS MEMBERS OF THE WISCONSIN ELECTIONS COMMISSION; MEAGAN WOLFE, IN HER OFFICIAL CAPACITY AS THE ADMINISTRATOR OF THE WISCONSIN ELECTIONS COMMISSION; SENATOR ANDRÉ JACQUE, SENATOR TIM CARPENTER, SENATOR ROB HUTTON, SENATOR CHRIS LARSON, SENATOR DEVIN LEMAHIEU, SENATOR STEPHEN L. NASS, SENATOR JOHN JAGLER, SENATOR MARK SPREITZER, SENATOR HOWARD L. MARKLEIN, SENATOR RACHAEL CABRAL-GUEVARA, SENATOR VAN H. WANGGAARD, SENATOR JESSE L. JAMES, SENATOR ROMAINE ROBERT QUINN, SENATOR DIANNE H. HESSELBEIN, SENATOR CORY TOMCZYK, SENATOR JEFF SMITH, AND SENATOR CHRIS KAPENGA, IN THEIR OFFICIAL CAPACITIES AS MEMBERS OF THE WISCONSIN SENATE,

Respondents,

WISCONSIN LEGISLATURE; BILLIE JOHNSON, CHRIS GOEBEL, ED PERKINS, ERIC O'KEEFE, JOE SANFELIPPO, TERRY MOULTON, ROBERT JENSEN, RON ZAHN, RUTH ELMER, AND RUTH STRECK,

Intervenors-Respondents.

PETITIONERS' APPENDIX IN SUPPORT OF RESPONSE TO REMEDIAL MAPS

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TABLE OF CONTENTS

CERTIFICATION BY ATTORNEY

I hereby certify that filed with this brief is an appendix that complies with s. 809.19(2) (a) and that contains, at a minimum: (1) a table of contents; (2) the findings or opinion of the circuit court; (3) a copy of any unpublished opinion cited under s. 809.23 (3) (a) or (b); and (4) portions the record essential to an understanding of the issues raised, including oral or written rulings or decisions showing the circuit court's reasoning regarding those issues.

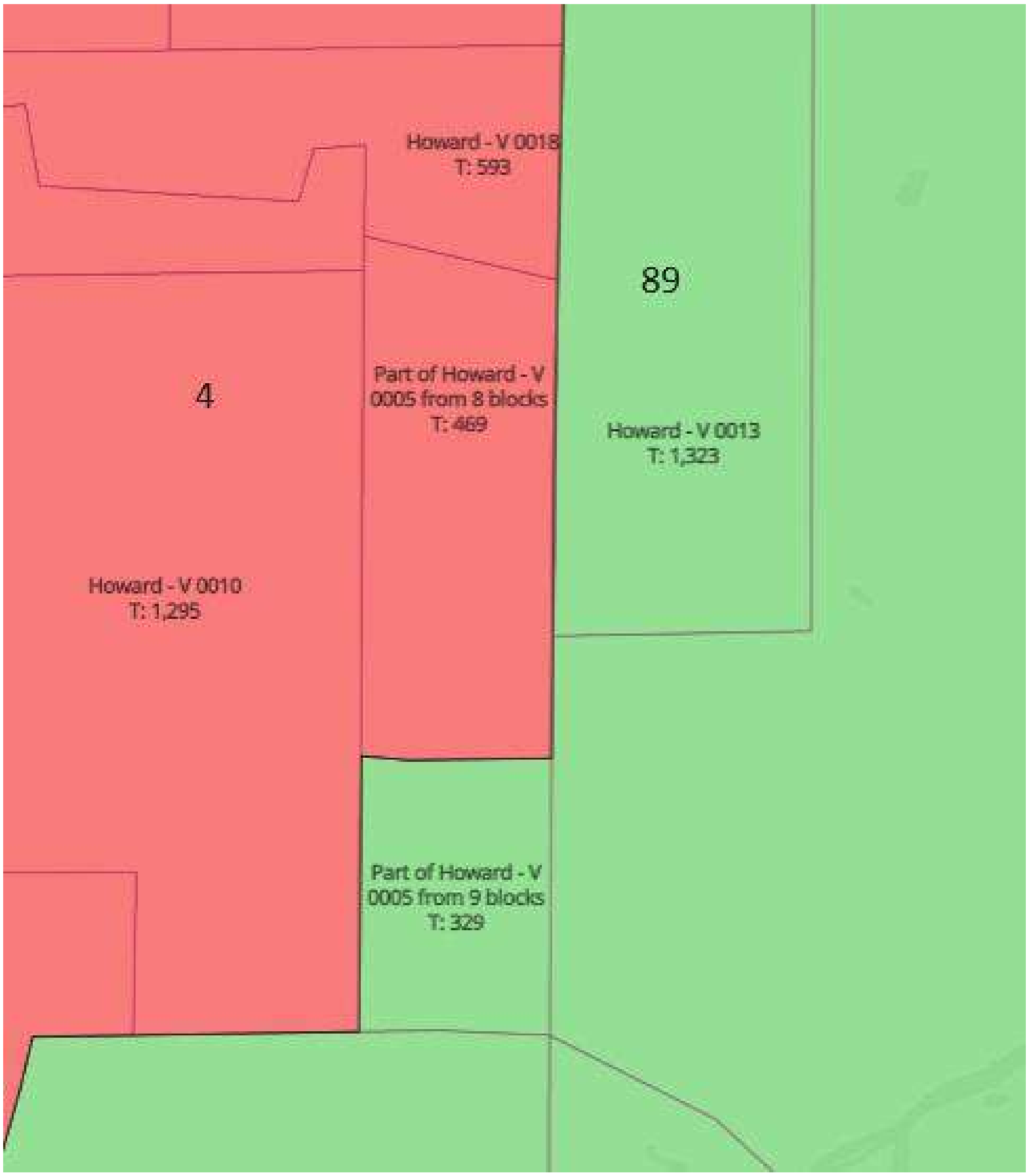
I further certify that if this appeal is taken from a circuit court order or judgment entered in a judicial review of an administrative decision, the appendix contains the findings of fact and conclusions of law, if any, and final decision of the administrative agency.

I further certify that if the record is required by law to be confidential, the portions of the record included in the appendix are reproduced using one or more initials or other appropriate pseudonym or designation instead of full names of persons, specifically including juveniles and parents of juveniles, with a notation that the portions of the record have been so reproduced to preserve confidentiality and with appropriate references to the record.

Electronically signed by Daniel S. Lenz
Daniel S. Lenz

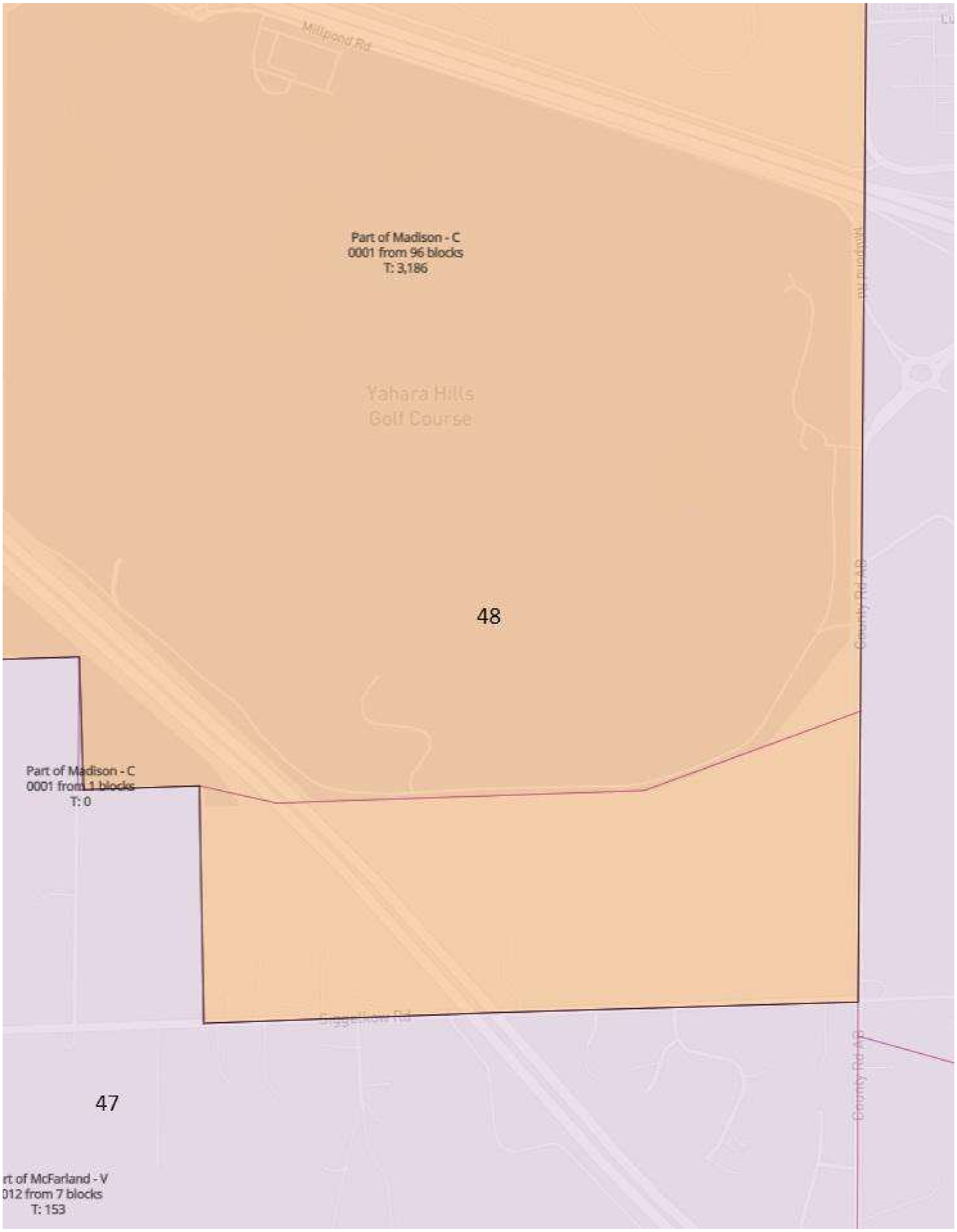
JOHNSON ASSEMBLY MAP WARD SPLITS ALONG DISTRICT BOUNDARIES[1]

Howard Ward 5 (AD4 & AD89)

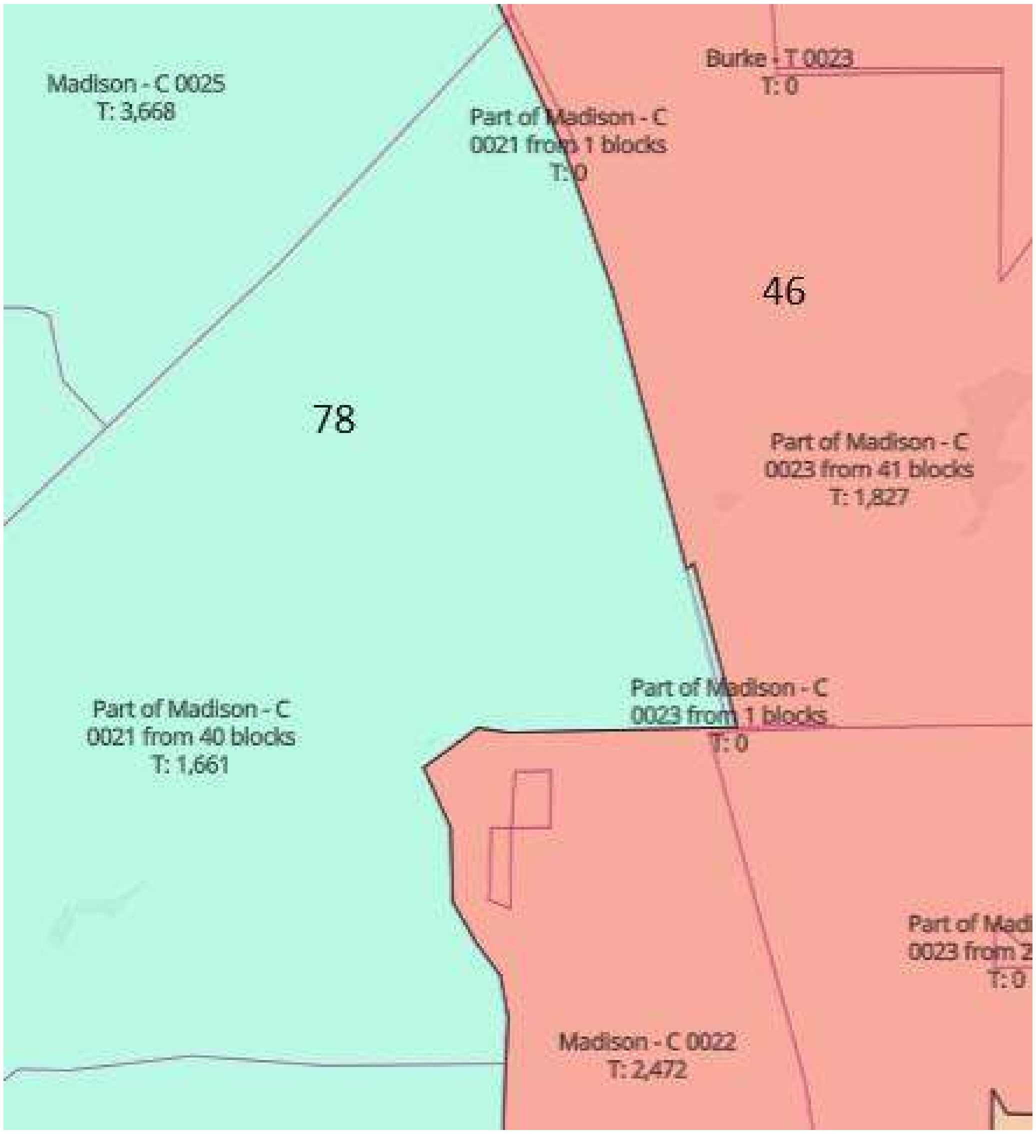


[1] These images come from the Dave's Redistricting App link provided by the Johnson Intervenors. *See* Johnson Assembly Map, https://davesredistricting.org/join/55a849c8-0687-4b89-ab78-b6b3c4e8097b. The Johnson Intervenors split another ward, former Town of Madison Ward 3, but that is a noncontiguous ward (that no longer exists) and the splitting of which does not affect the "bounded" provision as it occurs in the interior area of districts.

City of Madison Ward 1 (AD47 & AD48)



City of Madison Wards 21 and 23 (AD46 & AD78)



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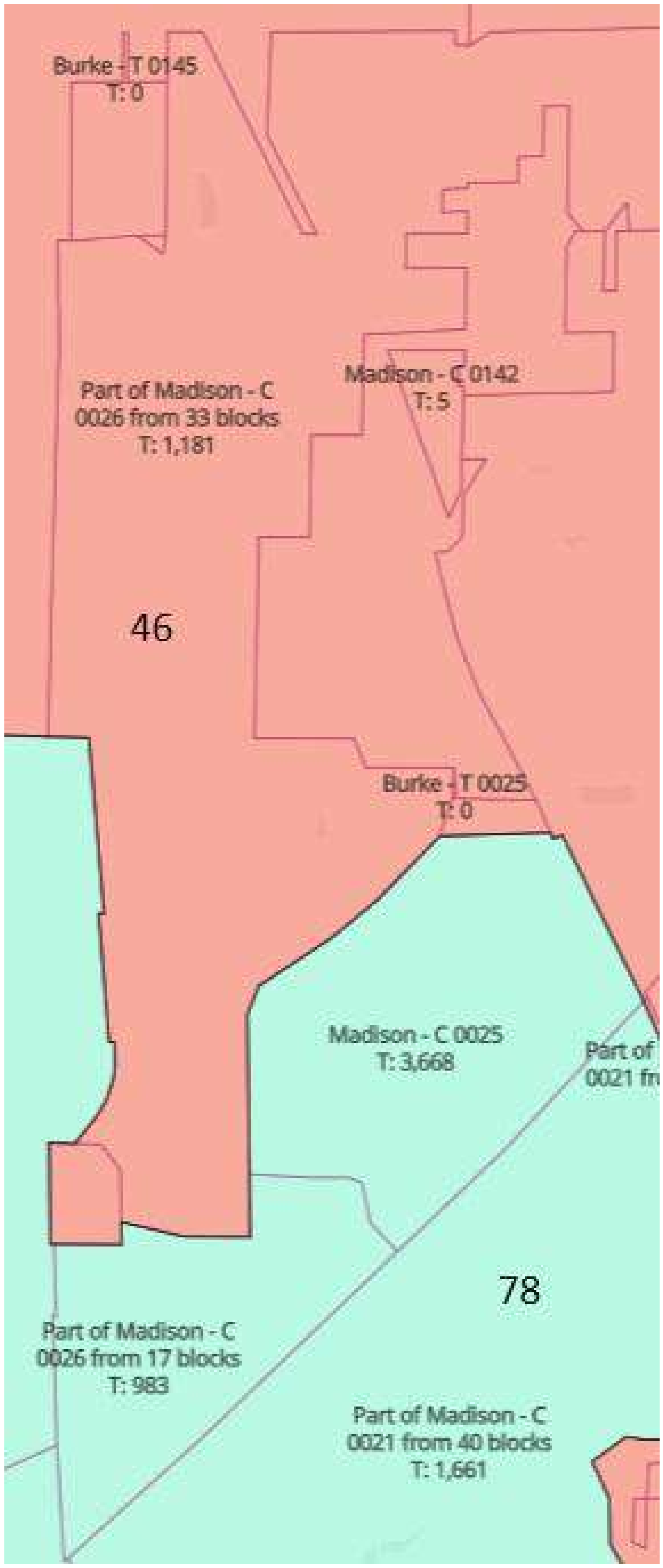
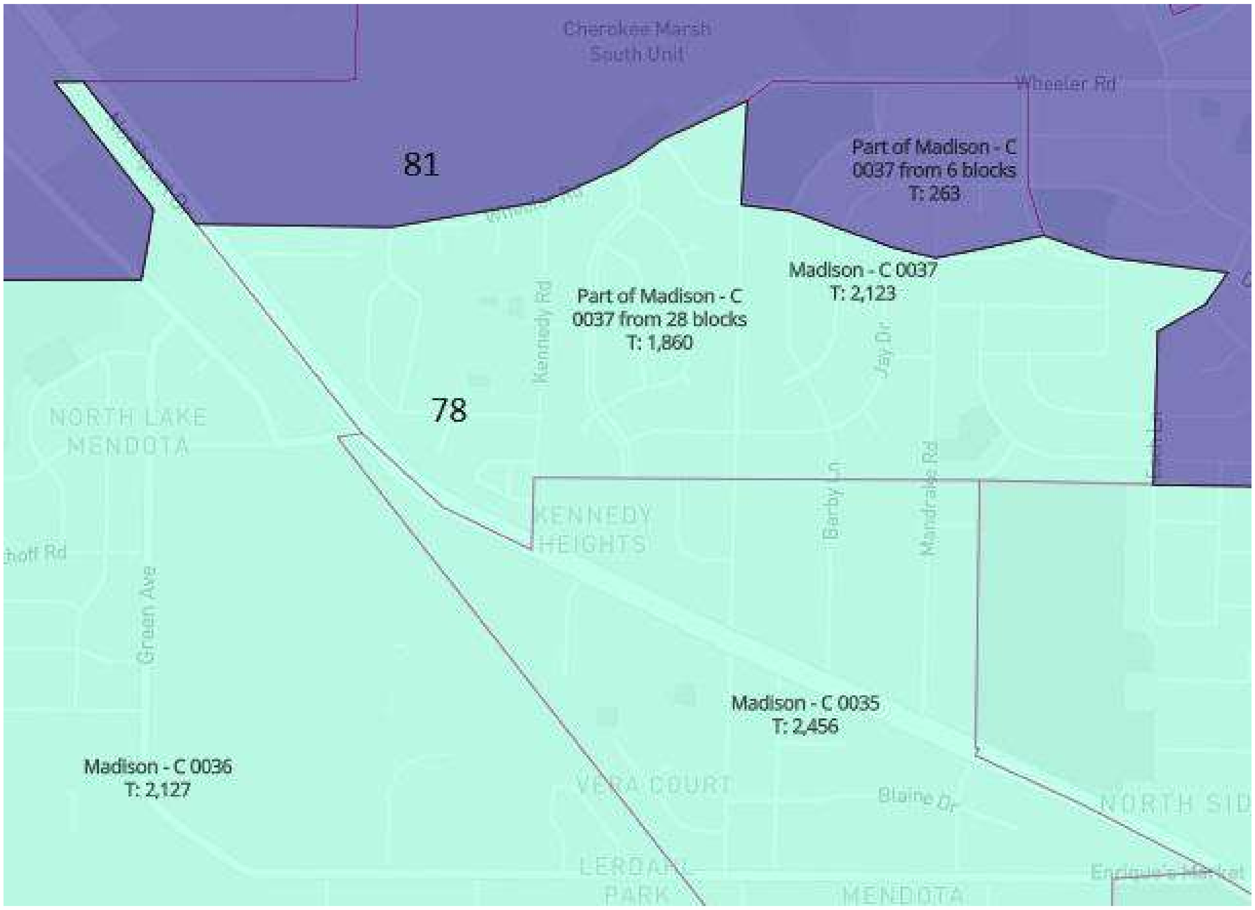
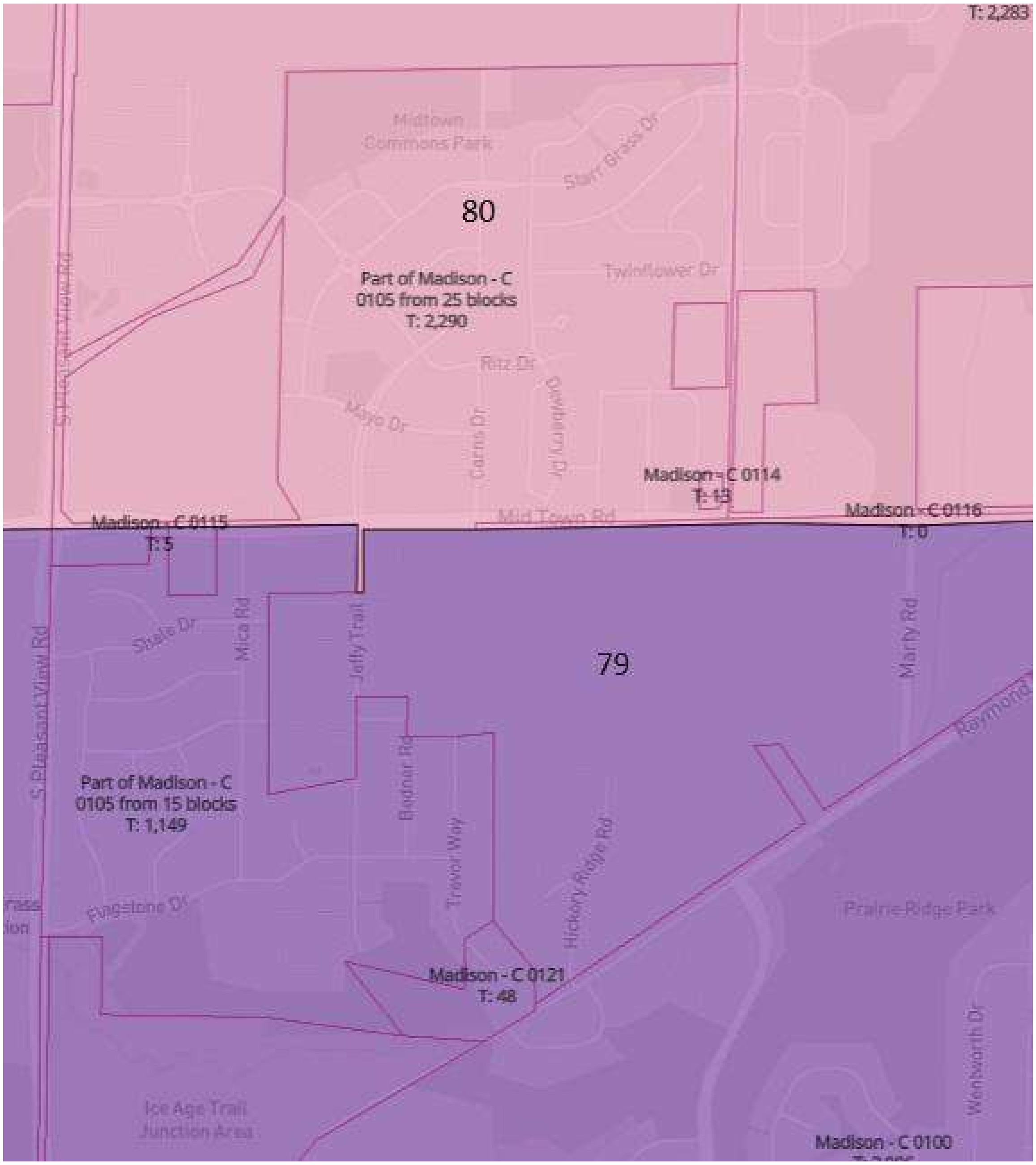


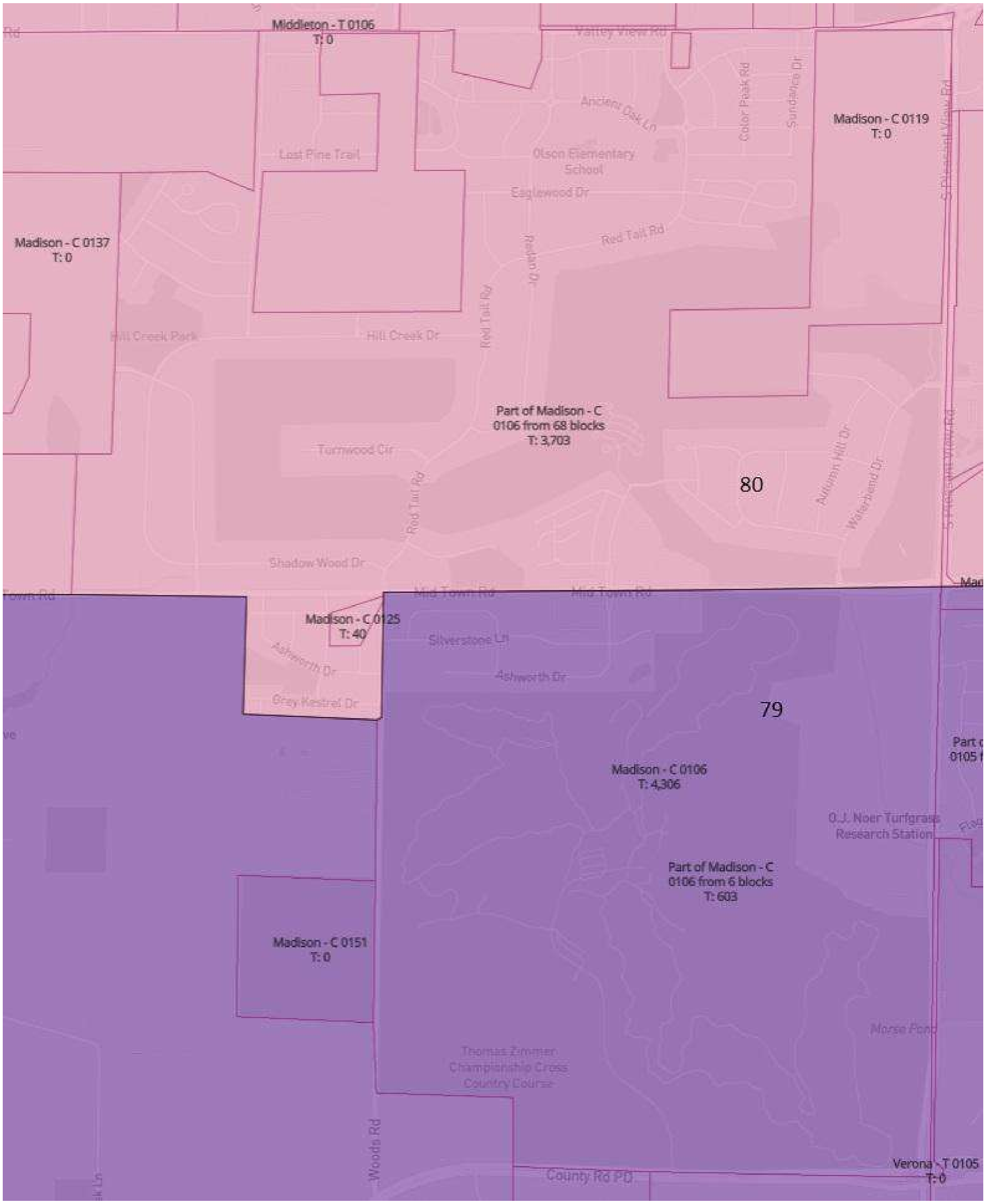
City of Madison Ward 37 (AD78 & AD81)



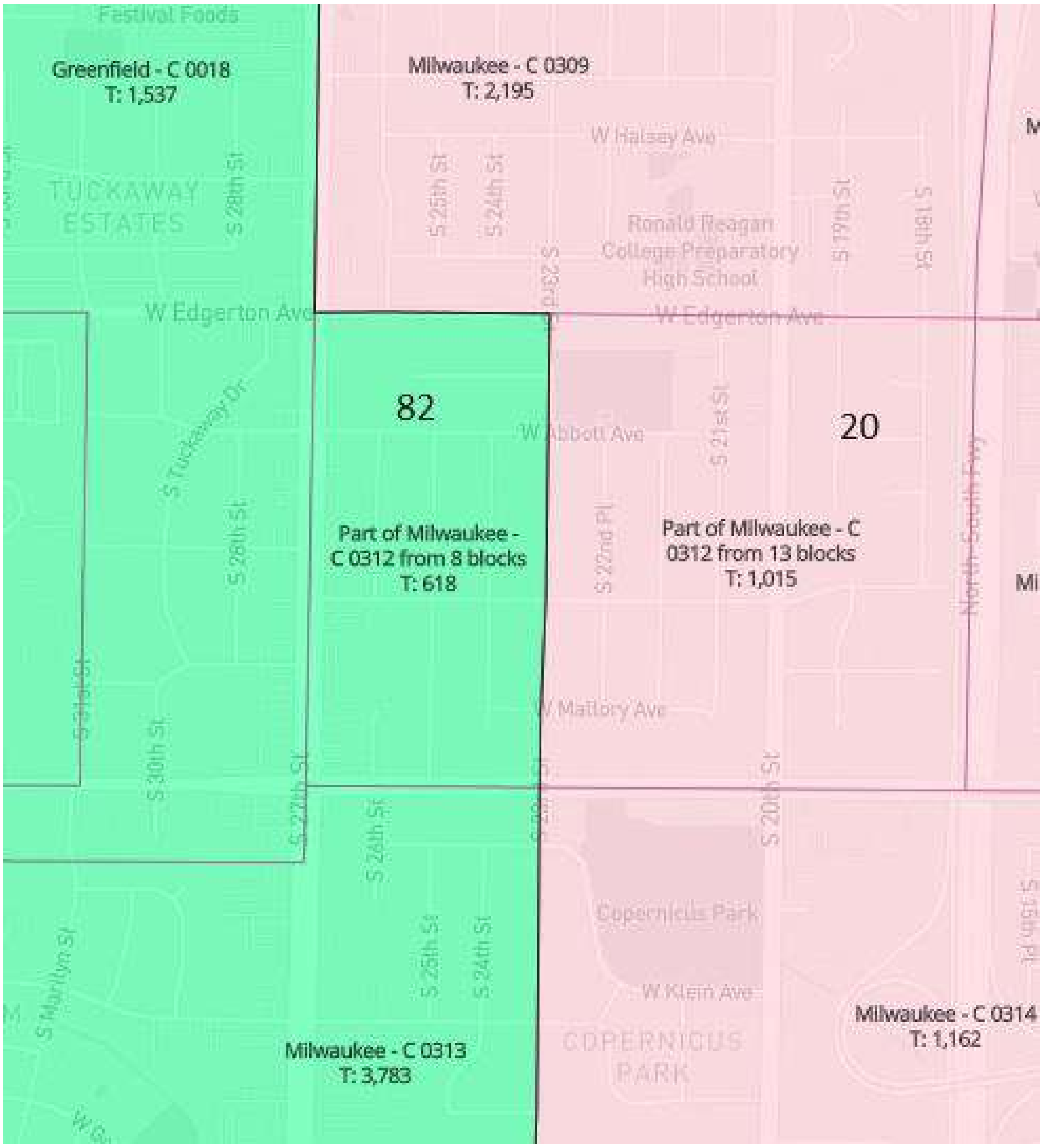
City of Madison Ward 105 (AD79 & AD80)



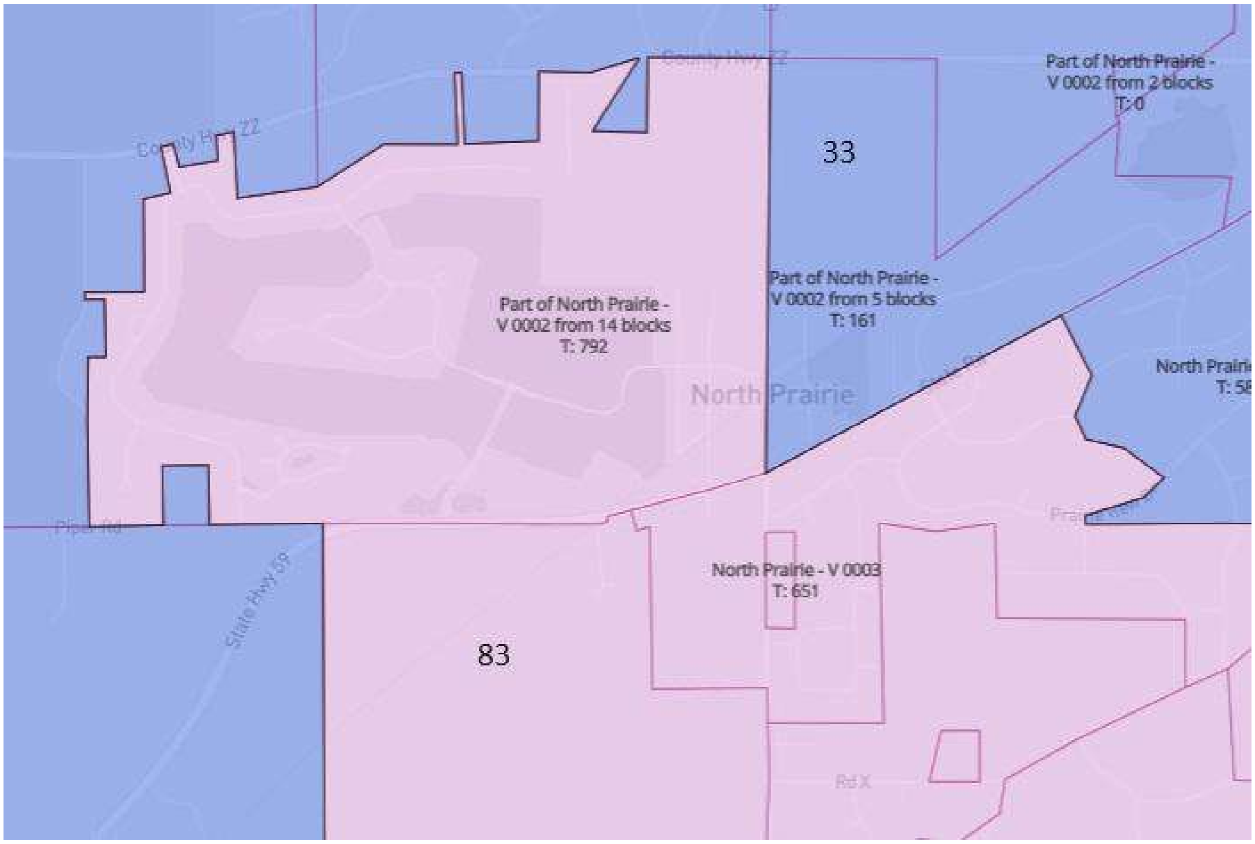
City of Madison Ward 106 (AD79 & AD80)



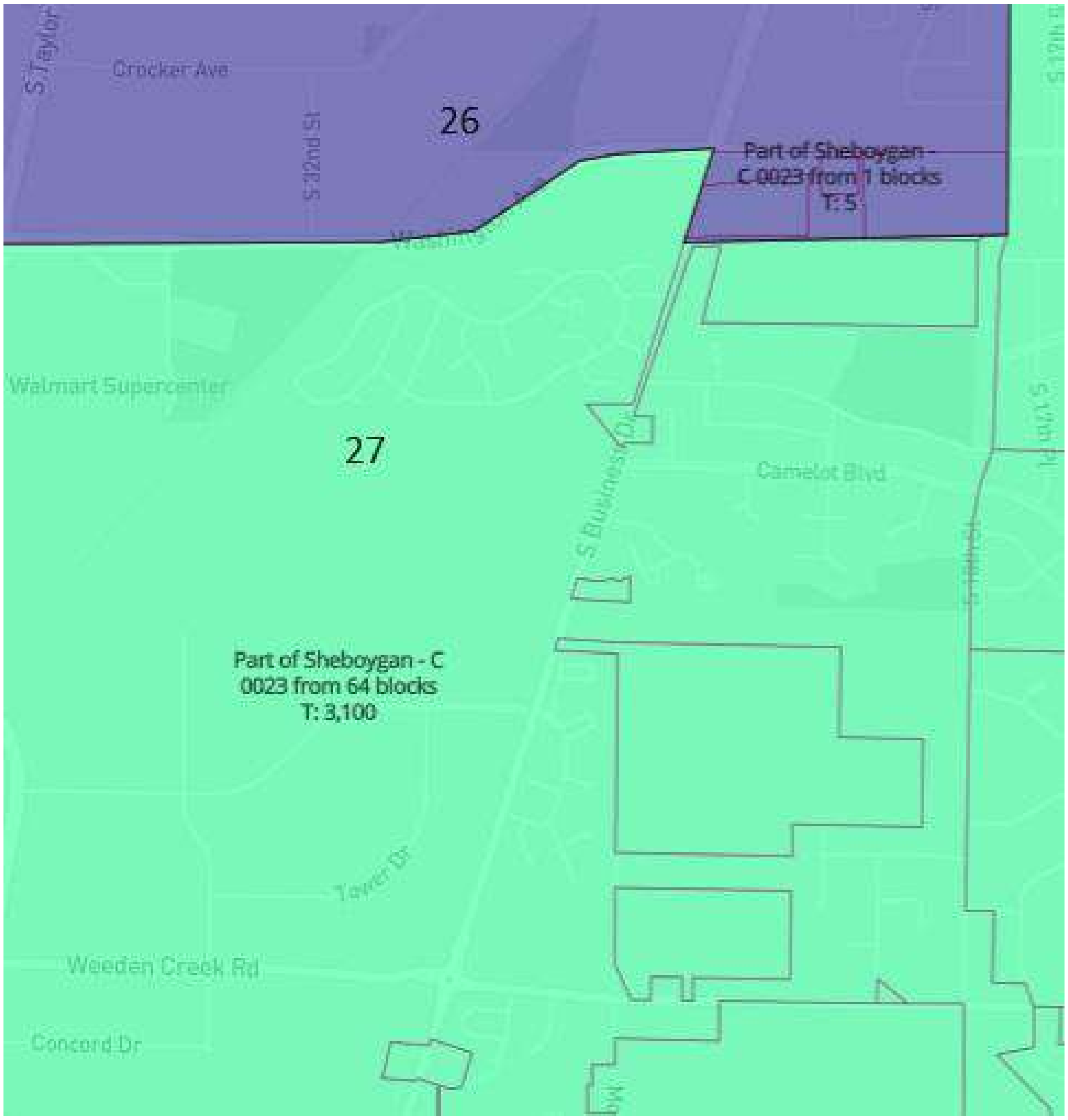
City of Milwaukee Ward 312 (AD20 & AD82)



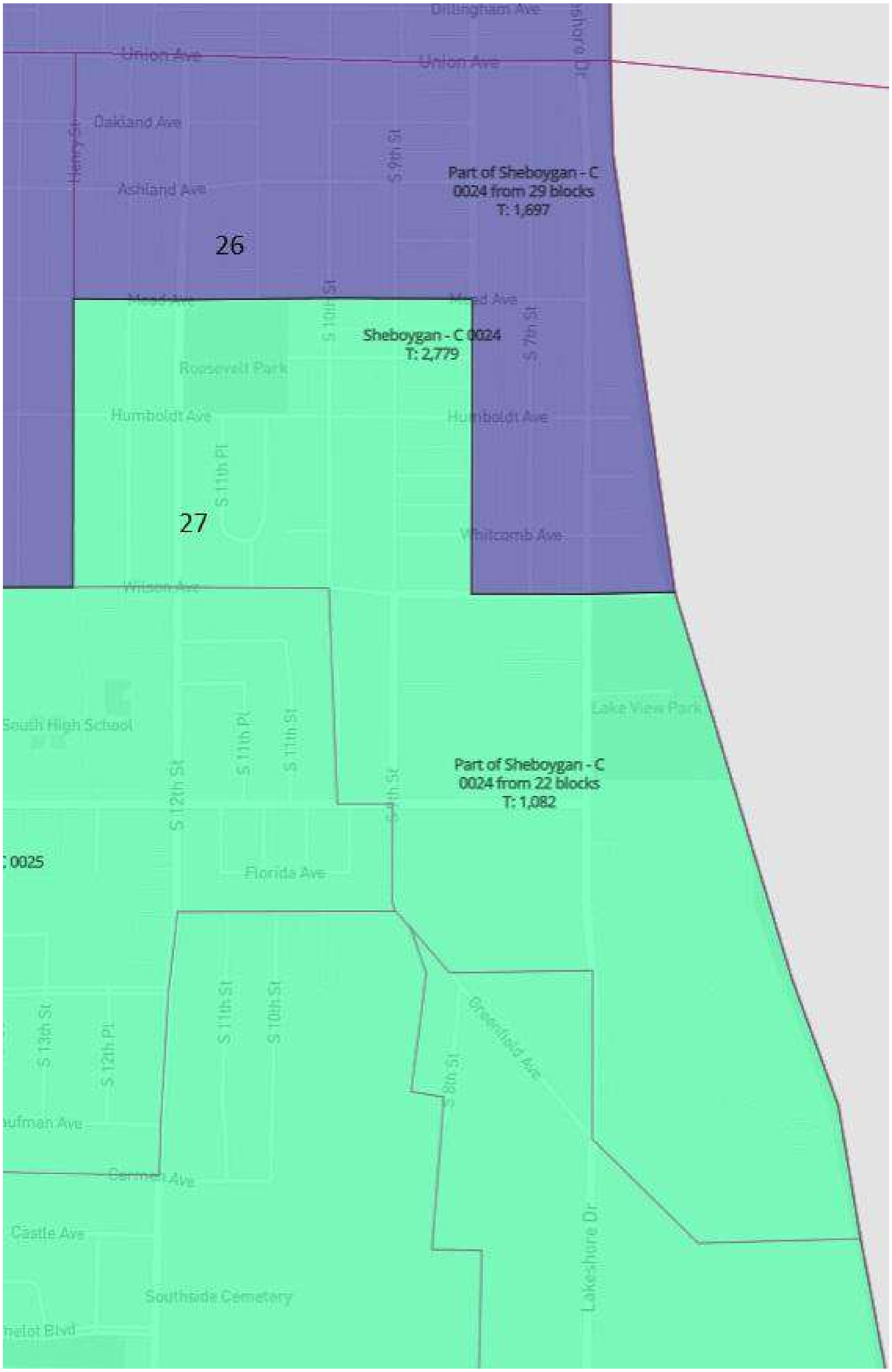
North Prairie Ward 2 (AD33 & AD83)



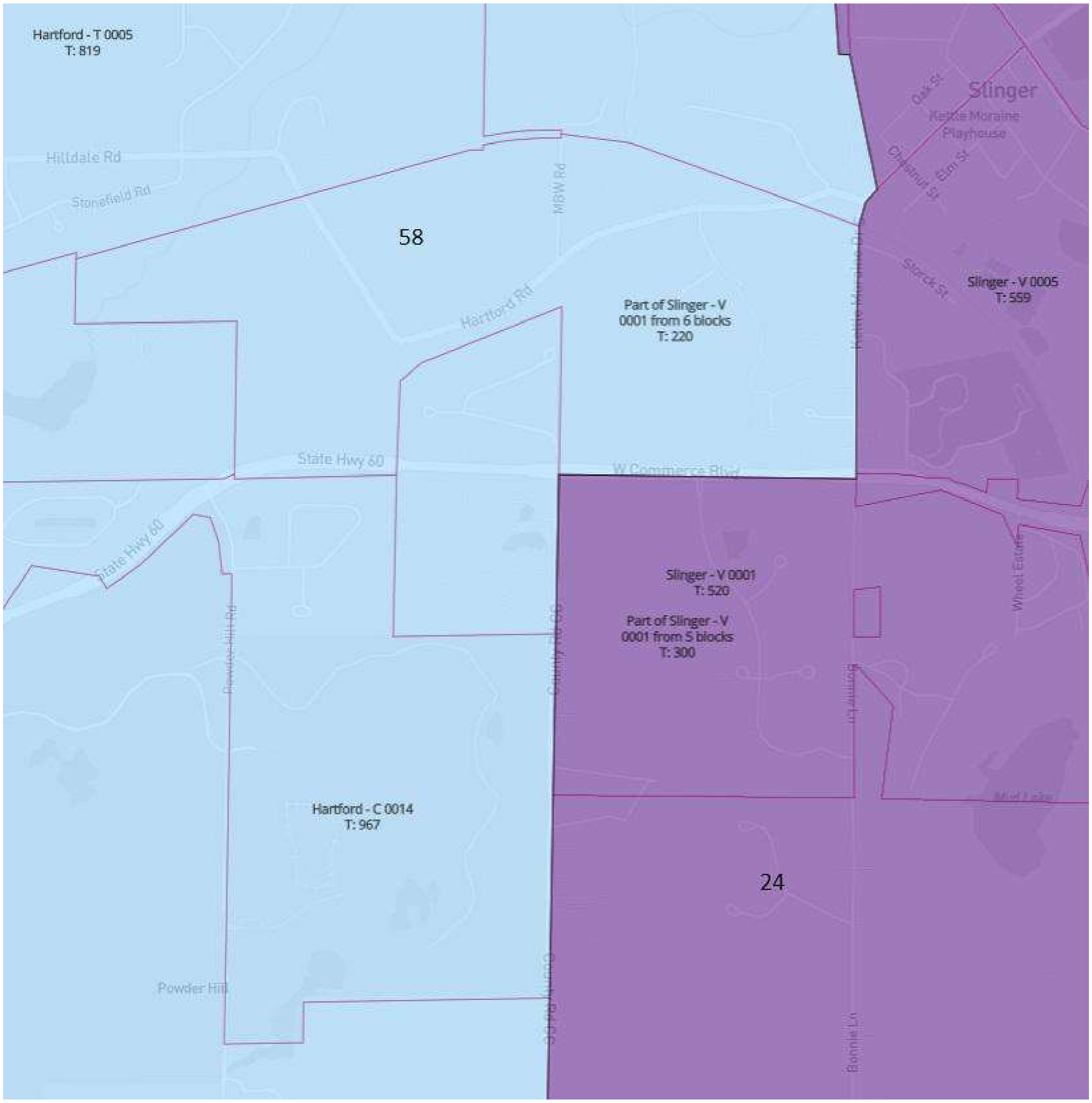
City of Sheboygan Ward 23 (AD26 & AD27)



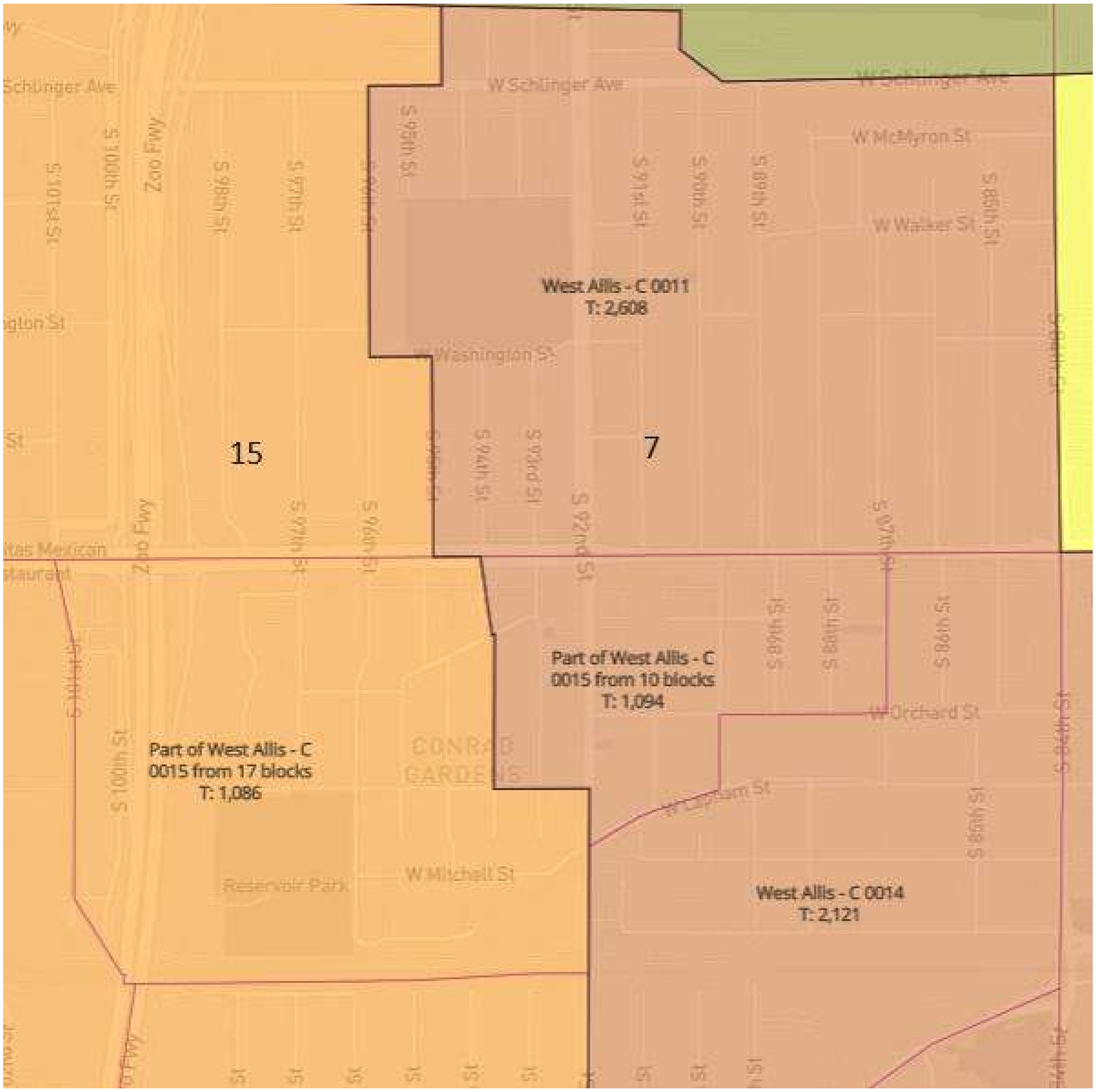
City of Sheboygan Ward 24 (AD26 & AD27)



Village of Slinger Ward 1 (AD24 & AD58)



City of West Allis Ward 15 (AD7 & AD15)



Preliminary Report:

Proposed Legislative and Congressional Remedial Plans in North Carolina

Revised draft (please discared the older version)

Bernard Grofman*

March 21, 2022

* I am Jack W. Peltason Chair of Democracy Studies and Distinguished Professor of Political Science at the University of California, Irvine. My research deals primarily with issues of representation, including minority voting rights and party competition. I am a Fellow of the American Academy of Arts and Sciences. I have an honorary Ph.D. from the University of Copenhagen for my work on the cross-national study of elections and voting rules. I am the recipient of a lifetime achievement award from the American Political Science Association for my work on elections and voting rights. I am co-author of five books with major university presses (Cambridge (4), Yale (1), and co-editor of 26 other books, (including books with Oxford (3), U. Michigan (4), and Princeton) with over 300 research articles and book chapters.. Over the past six years I have served as a special master to draw remedial maps for five different federal courts, including redrawing a Virginia congressional district and redrawing eleven districts in the Virginia House of Delegates, and preparing remedial maps s in local elections in Georgia, Virginia, and Utah. In addition I served as co-special master in the 2021 redistricting, drawing the remedial maps adopted by the Virginia State Supreme Court for that state's legislative and congressional districts. Over a 40+ year career, I have served as an expert witness or consultant in redistricting cases in nearly a dozen states I have worked as an expert for both political parties, the NAACP, MALDEF, the U.S, Department of Justice, and non-partisan redistricting authorities. My work has been cited in a dozen different U.S. Supreme Court cases, perhaps most notably in *Thornburg v. Gingles* 478 U.S. 30 (1986). In mid-February 2022 I was asked to serve as an expert consultant to the three Special Masters appointed to present recommendations to the North Carolina Supreme Court in the case of *Harper v. Hall*. North Carolina maps and block equivalency files were provided by the parties in this case; North Carolina election data was provided courtesy of the Voting and Election Science Team: https://dataverse.harvard.edu/dataverse/electionscience, disseminated by Dave's Redistricting App : https://davesredistricting.org of which I made extensive use.. I am also deeply indebted to my research assistant, Zachary Griggy, for the work he provided under my direction.

I. Introduction: Thinking About Partisan Gerrymandering.

We can address the questions of partisan or racial gerrymandering either directly in terms of observed or expected political or racial consequences or, more indirectly, by examining features of maps (e.g., undue fragmentation of existing political subunits) that are often manipulated for partisan purposes. In this report my focus is on political consequences.[1]

Another useful distinction in thinking about gerrymandering is whether the focus is to be on statewide indicators of gerrymandering or on evidence of gerrymandering at the district (or additionally, in North Carolina, county cluster) level. I believe in a holistic view of gerrymandering in which we examine both statewide effects and look in detail at evidence of manipulation at the level of districts/districts within clusters. Below I discuss both approaches.[2]

(1) Using statistical metrics to directly evaluate the degree to which a map as a whole is non-dilutive in its expected partisan (or racial) consequences?

Most analyses of partisan effects of gerrymandering rely on a set of measures in the political science literature such as the *mean minus median gap*, or *partisan bias* that are applied on a jurisdiction-wide basis. These two metrics are intended to be effectively independent of the actual state-wide vote share in any given election.[3] The mean-median gap builds in the value of the statewide vote average; by comparing means and medians of the partisan distribution, it is looking at one aspect of the skewness of a distribution, which is a measure of asymmetry. The partisan bias measure is evaluated in terms of what happens when both parties get a 50% vote share, and thus checks to see if one party is advantaged when the vote share is evenly divided at

[1] Since I have written extensively on racially polarized voting and racial vote dilution, if requested, I could extend my Report to analyze racial representation in the proposed maps. But, given the intense time pressure, I have limited myself here to issues involving partisan gerrymandering.

[2] Courts have differed in how they approached this issue. One possible synthesis is to evaluate maps at the jurisdiction wide level but to determine remedies in particular districts or particular areas of the state where the key problems seemed to lie. In the racial context, the finding of violations and the remedies for gerrymandering (or for a violation of the *Shaw v. Reno* 509 U.S. 630 (1993) test for a constitutionally unlawful racially preponderant motive) have usually been localized.

[3] However, *ceteris paribus*, both methods work best when, as in North Carolina, the state-wide two party vote share is close to fifty-fifty.

the statewide level. [4] Note also that the mean-median gap and partisan bias are NOT tests for proportionality; they are tests for unequal treatment.

The best known metric to evaluate partisan inequities is *partisan bias*, one measure of which is reported for proposed NC maps in Table 1 later in the Report.[5] The *partisan bias* metric, which focuses on what happens when the vote share is 50%, implicitly incorporates what Dr. Duchin in her first expert witness report refers to as the *majoritarian principle*, namely that a majority of votes should translate into a majority of seats. As the Supreme Court said in *Reynolds v. Sims*," to sanction minority control of state legislative bodies would appear to deny majority rights in a

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[4] Similarly, the difference between the value of the *efficiency gap* for a given plan and a value of the *efficiency gap* of zero can be taken to be an indicator of possible gerrymandering.

[5] The partisan bias test, based on symmetry, was developed by the Princeton political scientist, Edward Tufte in 1973 and the statistical methodology for calculating it was improved by the Harvard political scientist Gary King and his co-authors in the 1980s, mostly notably in joint work with the Columbia University statistician, Andrew Gelman. A relatively non-technical introduction can be found in Bernard Grofman and Gary King. "Partisan Symmetry and the Test for Gerrymandering Claims after LULAC v. Perry." 6 *Election L.J.* 2 (2007). Also see Katz Jonathan N., Gary King, and Elizabeth Rosenblatt. 2019. "Theoretical Foundations and Empirical Evaluations of Partisan Fairness in District-Based Democracies." American Political Science Review. Partisan has a simple intuition but requires a somewhat complicated method to generate results. Take a situation in which Democrats typically won approximately 53% of the statewide two-party vote. Say that with 53% of the vote Democrats would win 57% of the seats in some legislative or congressional election. Now, say that in a succeeding election, Democrats lost 6 percentage points in the popular vote so that they, not the Republicans had 47% of the popular vote. If the map were perfectly symmetric, with 53% of the vote, the Republicans also should win 57% of the seats, as the Democrats did with this same vote share. Calculating partisan symmetry requires that a researcher estimate a 50-50 election. In our example above~~,~~ the researcher begins with a 53% vote share and then shifts the vote share, on average, a point at a time in both the Republican and Democratic direction while tracking the expected outcomes in seats won and lost. Then the relationship between vote share and seat share is calculated. If the parties move identically up and down what is called a votes-seats curve, the deviations should cancel out and you are left with a 0 deviation from symmetry, i.e., an estimated seat share of 50% at a vote share of 50% (i.e., vote share of 50% at a seat share of 50%). If the outcome at a 50% vote share is something other than a 50% seat share then there is partisan bias in favor of one party or the other. While this metric can be time consuming to calculate by hand, a computer can calculate this quickly. Note that a 53% vote share need not require a 53% seat share for the map to be non-dilutive. Note also that we need to a test to see if the observed level of bias is statistically significant. If a large proportion of seats are competitive, then an estimated bias may not be statistically significant, since a small change in vote share in some of the competitive seats can shift seat share substantially. This metric is the only one to attract favorable mention by some Supreme Court Justices (see Grofman and

way that far surpasses any possible denial of minority rights that might otherwise be thought to result " 377 U.S. 533 at 565 (1964).[6]

While the mean-median gap is a very useful and easy to calculate tool for getting a handle on the presence of partisan gerrymandering, it cannot stand as the sole statistical measure of partisan gerrymandering. Not only does it need to be informed by the results other measures, such as partisan bias, but it also can usefully be supplemented by measures which extend its basic approach beyond a single district.

Dr. Duchin in her first expert witness trial report (PX150, Figure 2, at p.7) shows data for the enacted congressional map and congressional ensembles. and looks at the set of most competitive districts (not just at one district, the median district). She examines whether the set of competitive districts are skewed in favor of one party. She refers to this approach as the "close votes, close seats" principle. Analogous analyses are performed by Dr. Chen in his trial testimony (see PX482, pp. 30-31). This approach can be thought of as a generalization of the mean-median gap, and is arguably to be preferred to it, since the mean-median gap only deals with results for a single district and thus can present a misleading picture of the partisan consequences of a map as a whole. Also, the mean-median gap may be easier to manipulate by mapmakers than some other measures, e.g., by assuring that in the particular district which is the median, the mean-median gap is not that big even though the map as a whole remains a clear partisan gerrymander. Nonetheless, largely because of its simplicity, the mean-median metric is an important one. I have used it myself in evaluating maps when appointed in 2021 by the Virginia Supreme Court as co-Special Master for Virginia congressional and legislative redistricting.

But, regardless of which measure of partisan vote dilution is being used, it is important to also consider how likely to be durable is the gerrymandering effect. As the Supreme Court of North Carolina observed in *Harper v. Hall*. "While partisan gerrymandering is not a new tool, modern technologies enable mapmakers to achieve extremes of imbalance that, 'with almost surgical precision,' undermine our constitutional system of government. Indeed, the programs and algorithms now available for drawing electoral districts have become so sophisticated that it is possible to implement extreme and durable partisan gerrymanders that can enable one party to effectively guarantee itself a supermajority for an entire decade, even as electoral conditions change and voter preferences shift" (slip op., p.1, footnotes omitted).

(*2) Looking at evidence of partisan manipulation* <u>*at the district or county cluster level*</u>

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[6] The majoritarian principle is much weaker than the proportionality principle; the latter requires that a given vote share for a party translate into the identical share of legislative seats for that party. My 1985 essay, "Criteria for Districting: A Social Science Perspective." <u>UCLA Law Review</u>, 33(1):77-184," is among the many which discuss the importance of the majoritarian principle for democratic theory and election law

To look for evidence of gerrymandering at the district or county cluster requires an intensively local appraisal of how political subunits, concentrations of voters of a given party, and demographic groups are being treated (as well as of the degree to which compactness concerns were being met). This can be accomplished in two different ways.

One way is to look for evidence about intentional manipulation of boundaries at the district or county cluster level by careful use of the eyeball (and perhaps also some simple descriptive statistics) by individuals who have detailed knowledge of the state and who then provide a description of how particular pieces of geography were manipulated. Here, we can either be looking to identify areas where gerrymandering is found and to which remedies might be directed and/or we look for ”patterns and practices” that are common across subunits of a kind that are indicative of gerrymandering even if we do not formally test for statistical significance[7] This type of common-sense evidence can be compelling, both at the level of individual districts and for understanding an overall pattern of dilutive acts.

The second way is to make use statistical analyses for districts or county clusters is to do analyses based on ensembles in ways that closely resemble those used for statewide analyses.

For example, one useful approach to understanding the degree to which the two key tools of gerrymandering, *packing* and *cracking,* were used by mapmakers at the district level employs ensemble analysis and calculation of statistical outliers. Dr. Jowei Chen in his expert witness trial report. Dr. Chen (PX882, Figure 4, p. 25) ranked congressional districts from most Republican to least Republican in the enacted congressional map, and considered whether there was evidence of manipulation in that the districts Republicans did best in were, in general, being won by lower than expected vote margins (i.e., the map “efficiently” placed Republican voters to win without wasting Republican votes), while the districts in which the Democrats did best were, in general, being won by higher than expected vote margins (i.e., the map “inefficiently” placed Democratic voters to “pack” them and thus waste their votes), while districts that were somewhat competitive by and large showed a higher than expected Republican votes hare (those districts were “shored up” to make Republican loss unlikely).This creates an s-shaped pattern in the data that is clearly visible in Figure 4. [8] This type of evidence suggests, even if it cannot prove, intentional partisan gerrymandering,

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[7]*Descriptive statistics* simply describe data and patterns in the data; *inferential statistics* seek to assign probability of occurrence of events relative to some null hypothesis. With ensemble analysis, the null hypothesis against which statistical significance is determined is that the plan was drawn from a set of plans like those in the ensemble.

[8] Chen observes statistically significant results in 10 of 14 of the county clusters and the overall pattern is striking. Here it is important not to be misled by the fact that there were some clusters that were not statistically significant; it is the overall pattern that shows the improbability of the results. Indeed, even if there were NO clusters with statistically significant results but the directionality of manipulation was as predicted across virtually all the clusters, properly applied statistical calculations that look at multiple clusters at the same time can show the reality of statistically significant results even if no single cluster is a statistically significant outlier. [8]

At the county cluster level, we can also evaluate whether there were excess city splits or county cuts within that cluster from what we would expect of plans in the ensemble in that same clusters. We should also note that we can ask if expected partisan outcomes within the cluster in terms of mean expected wins were extreme statistical outliers, or whether particular groups such as African-Americans or other minorities were either cracked or packed within the cluster in ways that signaled improper attention to race. But we must be careful not to mistake failures to find statistically significant results at the cluster level with the absence of significant (and substantively important) bias in the plan as a whole, since what is a clear overall pattern of discrimination can be missed if we look only small groupings.

But, in looking at districts or clusters, just as in looking at stateside indicia of partisan gerrymandering, we must also ask whether difference from what is predicted in an ensemble takes us toward partisan equity or away from it (see below).

II. Baselines and Thresholds in Evaluating Partisan Gerrymandering

What is the appropriate baseline against which to judge whether some given value of a metric such as partisan bias or mean-median difference supports a claim of egregious gerrymandering?

There are two ways in which the question of appropriate baseline for statistical analyses of partisan gerrymandering effects has been addressed in the political science literature. The most obvious way to evaluate statistical metrics used to identify partisan gerrymandering effects, such as those shown in Table 1, is simply to ask questions such as: "How close is the mean-median gap to <u>zero</u>?" "How close to a <u>zero</u> level of (vote or seat) partisan bias does the plan have?, etc. As a result of my recent experiences as a special master I have come to the view that this is not just the simplest, but also the best, way to think about statistical metrics that seek to directly measure gerrymandering. But a second way in which this question has been addressed is to ask: "How does a map compare in its properties vis-a-vis various metrics to those in an ensemble of computer drawn maps constructed in a partisan blind fashion?"

Ensembles are sets of computer-generated plans based on the geographic distribution of population in the unit (usually at the level of census blocks) which may also have "built in" instructions to the computer to take into some features besides population, e.g., respecting county or other subunit borders, or avoiding pairing incumbents, or seeking to draw compact districts.[9] For ensembles, for any given metric, the baseline is established by answer the question: "Is a given map a statistical outlier with respect to the ensemble, with properties that by chance alone would occur only at the tails of the ensemble distribution, e.g., with probability less than .05 (the familiar two standard deviation test for *adverse impact* from *Griggs v. Duke Power Co*, 401 US

[9] In North Carolina, ensemble simulations for state legislative districts (NC House and NC Senate) introduced by experts in *Harper v. Hall* are programmed to take into account, the state's county clustering rule.

424 (1971))?" Ensemble analysis can be applied to features of maps such as splitting of counties or other subunits, or features such as compactness, but it can also be applied to measuring expected political effects of a map via the kinds of metrics used by experts in the *Hall v. Harper* litigation, that were subsequently referenced in the *Harper v. Hall* majority opinion.[10] Election-related metrics are calculated using a distribution of recent partisan (and/or racial) voting patterns in the unit (usually with data drawn from statewide elections that is projected into census geography). with the values of these metrics and of expected partisan outcomes in the plan (or portions of the plan) are compared to those in the ensemble.

In evaluating any map in terms of political effect metrics it is important to be able to separate out the effects of so-called "natural" bias, i.e., partisan bias that arises from historical patterns of electoral geography and environmental features such as mountains or rivers,[11] from partisan bias that arises from contemporaneous map-making practices, including and especially intentional gerrymandering. Using ensembles as the basis for our evaluations directly allows us to compare the bias (or other features) in any given map with the bias (or other feature) in the ensemble, since we are holding constant the electoral geography of the state and other features of the state, such as rivers or mountains.

The use of ensembles has allowed for major theoretical and empirical advances in studying redistricting and gerrymandering, and I strongly endorse their previous use in this litigation. If a map exhibits more evidence of bias or other kinds of distortions than we find in an ensemble to a statistically significant degree, I view this fact as very strong *prima facie* evidence of manipulation. But there are two ways to make errors based on ensemble analyses involving political election-based metrics: on the one hand, concluding that a plan is dilutive when instead it is vote-dilution reducing and, on the other hand, concluding that a plan is not dilutive because it is not an outlier in the ensemble for some parameters when, in fact, it is a carefully crafted gerrymander (Type I and Type II errors).

First, we must be careful to look at the directionality of deviation from ensemble expectation. **If a map has lower (absolute) values on metrics such as partisan bias than most of the maps in the ensemble, *ceteris paribus*, that is something to be desired, not condemned, even if the map is outside the 95% confidence range of the ensemble. It is only when the map** has higher values of metrics that show vote dilution than most of the maps in the ensemble that we see evidence of partisan gerrymandering that might rise to the level of unconstitutionality. Thus, even if we opt only for an ensemble based approach to evaluating vote dilution, when we do look at how far from an ensemble expectation is the observed value on some metric it is critical to

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[10] See, e.g., the discussion of the findings of Plaintiffs expert Dr. Jowei Chen in *League of Women Voters v. Pennsylvania* (J-1-2018, Supreme Court Of Pennsylvania Middle District).

[11] Although I have used the term "natural bias" because it has become standard, I regard it as a misnomer. For example, there is nothing natural about the disproportionate presence of African-Americans in areas good for cotton growing that continues to the present day, unless you think slavery is natural. And redlining and other practices have led to geographic segregation of minorities within cities.

distinguish whether the value in the map takes us in the direction of more dilution or in the direction of less dilution.

Second, **even if a map is within the 95% confidence bounds of an ensemble on some particular metric, that does not mean that the map is NOT a partisan gerrymander**. There are multiple statistical metrics to evaluate the level of partisan gerrymandering, and we need to be careful to look at multiple indicators, both at the state level and ones that are district or county-cluster specific. Also, there may be non-statistical evidence of intentional gerrymandering derived from careful analysis by knowledgeable observers of exactly where particular lines on the map have been drawn. Such evidence may lead to a conclusion of a constitutional violation even in the absence of use of ensembles or of statistical inference tests.[12] Or they may be inferences of intentional gerrymandering based on the redistricting process itself or based on statements made by mapmakers.

Third**, because of how ensembles are created, when we look at the political effects metrics, they may show a map to be non-dilutive even when dilution is present because the natural bias in a state favors a particular party and thus tilts the ensembles toward maps favorable to that party.**

An ensemble-based standard for vote dilution takes as given the distribution of voters in the state at some low level of census geography such as the block. But because it is built on the distribution of voters, when we look at partisan behavior in past elections, we often find that the voters of one party are more concentrated than voters in the other party. In particular, Democrats (and minorities) are likely to be highly concentrated in cities. When one group has its voters more geographically concentrated than another, redistricting can create inequities, e.g., by packing Democratic voters into districts in such a fashion as to "waste" their votes.

While I can attest from my own knowledge that Dr. Duchin (PX150, p. 4) is correct that North Carolina is a jurisdiction that has a low level of so-called natural bias compared to most other states, [13] a low level of natural bias is not zero bias.

Consider the ensembles created by Dr. Daniel Magleby which he uses to evaluate whether some given plan's mean-median value is (considerably) outside the 95% confidence range generated by the ensemble (see PX 1483). For Congress, Magleby finds the mean-median value in his ensemble to be around 1% more Republican than the statewide average (see Figure 5 in his first Report). A similar 1% pro-Republican bias is found for the Senate (see Figure 4 in his first

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[12] Much of the litigation involving claims involving racial gerrymandering or race as a preponderant motive illustrates this point.

[13] The existence of what has been called "natural bias," has led some commentators to claim that whatever bias is found in a given plan is due to geography, not intent to discriminate. However, as Dr. Duchin correctly points out, the level of natural bias in North Carolina in no way prevents the production of "maps that give the two major parties a roughly equal opportunity to elect their candidates" (PX150, p. 4).

Report), while the pro-Republican bias in the House for the mean-median ensemble is between 2% and 3%.(see Figure 2 in his first Report)

Further evidence of a pro-Republican "natural bias" obtain from simulations that focus on the expected number of seats a party will be expected to get if the partisan vote share is at the historical recent average. Dr. Magleby has done analyses of this kind (see PX1483), but so have other plaintiffs' experts. For example, with a projected 50.8% Republican vote share, while the 10-4 projected vote outcome in the 2022 enacted legislative congressional map is a clear statistical outlier, Dr. Chen finds that a modal congressional outcome in his simulation would have an expected 9 Republican and 5 Democratic seats for the U.S. House (see Report of Dr. Chen PX882, Figure 7, p. 33). Dr. Mageleby's simulation (Figure 6 in his first Report) is similar, with about 8-9 Republican seats.

In sum , so-called "natural bias" tilts the ensembles for the North Carolina upper and lower chambers and for the U.S. House of Representatives somewhat in a pro-Republican direction.[14]

Resting analyses of partisan bias solely on outlier analysis in ensembles creates a two-sided risk. On the one hand, **plans that are highly dilutive might be accepted if the only analysis of equal treatment is an ensemble-based comparison**. **Indeed, <u>if</u> we judge partisan outcomes only by whether they closely match the mean results in an ensemble, we might conclude that, in North Carolina, for both branches of the legislature and for Congress, only at least a somewhat pro-Republican gerrymander is non-dilutive.**[15] On the other hand, any attempt to move toward a truly unbiased map might require moving away from the level of bias that is created by geography, i.e., outside the middle zone of the ensembles, and thus be attacked as a gerrymandering outlier. Such perverse results would, in my view, fly in the face of the North Carolina Supreme Court's assertion that "We hold that our constitution's Declaration of Rights guarantees the equal power of each person's voice in our government through voting in elections that matter" (slip op. p.1).

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[14] As best I can judge all the ensembles created by plaintiffs' experts show an expected pro-Republican tilt in partisan effects measures such as mean-median difference.

[15] The ensemble analyses conducted by Plaintiffs experts in *Harper v. Hall* concluded that the enacted maps to be partisan gerrymanders in that these maps were so egregiously gerrymandered that, on multiple indicators, they fell very far outside the ensemble-based expectations of the amount of expected pro-Republican bias even though the computer-generated ensembles were themselves exhibiting a pro-Republican bias (see above). The ensembles-based conclusions that these maps were egregiously gerrymandered in favor of Republicans, combined with the other evidence of intent and examination of how gerrymandering was done in particular areas of the state, combined with the evidence that the extreme level of pro-Republican bias in these plans would continue throughout the decade under realistic scenarios of future changes in statewide vote, thus locking in a permanent Republican majority in both houses of the legislature and in the state's congressional delegation, made it apparent that the plans should have been struck down as unconstitutional once partisan vote dilution was held to be justiciable under North Carolina state law.

Can we specify some threshold value of a metric such as partisan bias or mean-median difference as being required to supports a claim of egregious gerrymandering that rises to the level of unconstitutionality ?

Both the zero baseline approach and the ensemble-based approach still leave open the question of the point at which the accumulated evidence of gerrymandering leads to a conclusion that this gerrymandering rises to a level of unconstitutionality. But there is one question on which I think there would be widespread agreement, namely that a legislative map does not have to be the "best possible map." The mere fact that a better map on multiple criteria exists does not require a court to choose that map over a map that is adopted through legal channels and due process. The Court's role as mapmaker only begins after the challenged map has been found to be unconstitutional and the legislature has forfeited any right to continue to prepare alternative maps. Moreover, if we think about criteria for demonstrating unconstitutional partisan gerrymandering, there probably also would be agreement that (a) the mere fact that the value of on some metric is a statistical outlier is not enough to show a violation, rather there must be evidence of the substantive importance of the discrepancy,[16] and (b) before a finding of a constitutional violation, it would be important to demonstrate that the political effects of a plan are likely to be non-ephemeral.

However, while it might be seen as desirable for courts to clearly set a threshold for what differences from zero for any given metric are *de minimis* with respect to a claim of unconstitutional partisan gerrymandering, there are two reasons to reject such an approach at this time. First, state courts are only recently come to grips with partisan gerrymandering claims brought under state law. There simply has not been time enough for a body of jurisprudence to emerge. Rather, as the Court Opinion in *Harper v. Hall* suggested, courts should strike down egregious examples of partisan gerrymandering. Only in later cases will courts be in a position to set clear "safe harbor" thresholds if they eventually determine, as the U.S. Supreme Court did in the "one person, one vote" cases, that numerical *de minimis* standards were appropriate.[17]

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[16] In the context of redistricting, this would translate as a finding that the consequences of the statistically significant disparate impact involved an expected seat share change of, say, at least one district (though that number might vary with the size of the legislature). For example, in *League of Women Voters v. Pennsylvania* (slip op. p. 128) the Pennsylvania Supreme Court favorably cites to Dr. Jowei Chen's finding that " while his simulated plans [the ensemble] created a range of up to 10 safe Republican districts… , the 2011 [enacted] Plan creates 13 safe Republican districts."

[17] There are multiple statistical measures of malapportionment such as *total deviation*, defined as the sum of the deviation from ideal in the largest district plus the deviation from ideal in the smallest district, and *average deviation*, among others measures (see e.g., Cervas, Jonathan R., and Bernard Grofman. 2021. Legal, political science and economics approaches to measuring malapportionment: The U.S. House, the Senate, and the Electoral College 1790-2010. *Social Sciences Quarterly*. 101(6): 2238-2256), but, after a while, the Supreme Court largely settled on total population deviation as the key metric for OPOV. In the OPOV cases, after dealing with "horribles," The US. Supreme Court eventually adopted a 10% total population deviation safe

Second, ascertaining the level of gerrymandering in a map is harder than ascertaining the degree of malapportionment in a map. Not only are some of the statistical tools, such as ensembles, much more complicated than simple arithmetic but, perhaps even more importantly, there are multiple (but related) metrics and multiple factors to consider, all of which require careful parsing in terms of forging an overall assessment. Thus, I see the early phases of state court partisan gerrymandering litigation employing a "totality of the circumstances approach," even though also relying on the various specific statistical indicators the *Harper v. Hall* opinion highlighted. [18]

III. Preliminary Evaluations from a Political Science Perspective of the New Legislatively-Drawn Maps for Congress, the NC Senate, and the NC House

Below is a table showing, for each of the five proposed plans and for the three previously enacted maps, a variety of metrics: projections of how many Democratic and Republican leaning seats would be expected and how many districts would be competitive (from 45% to 55%) and also, among the competitive seats, what is the relative balance of Democratic and Republican vote shares; the mean-median gap; two standard measures of partisan bias based on symmetry in a seats-votes curve (one based on how much above a 50% vote share the party with diluted votes would need to win a majority of seats, the other based on the seat share a minority party would get if it won 50% of the vote); the efficiency gap; and a composite measure of compactness that incorporates Polsby-Popper and Reock scores. The calculations are provided from a program, Dave's Redistricting App, which can calculate the standard election-based indices of partisan gerrymandering. The political data reflect major statewide races 2016-2020. The metrics used give a historical baseline of 49.4% Democratic two party vote and 50.8% Republican two-party vote.[19]

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harbor for legislative districts – at least absent evidence of discrepancies lacking a legitimate state purpose, but required population deviation as close as practicable to zero for congressional maps. Having read the OPOV cases and gone back to read key academic commentary both just before and just after *Baker v. Carr*, I think it fair to say that nobody could have predicted the final OPOV standards chosen .

[18] *Brnovich v. Democratic National Committee* 594 U.S. ___ (2021) makes it clear that, in federal jurisprudence, in the context of Section 2 of the Voting Rights Act, a finding of disparate impact is not sufficient, standing alone, to prove a Section 2 violation, since other factors need to be taken into account, the U.S. Supreme Court also asserted "§2 does not transfer the States' authority to set non-discriminatory voting rules to the federal courts." This observation is doubly relevant, in my view, to the present litigation. On the one hand, the Supreme Court recognized the power of the states to set non-discriminatory voting rules. On the other hand, the Supreme Court recognized that no single metric may be enough to prove (or disprove) a constitutional violation, and that contextual analysis is needed.

[19] There is no dispute among experts that, in Dr. Duchin's words, "North Carolina voting has displayed a partisan split staying consistently close to even between the two major

<<Table 1 about here. See below>>

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parties over the last ten years." (PX150, p.4).

TABLE 1: Plan Comparisons on Multiple Metrics

Plan Name	Map Type	# of Districts	Rep Districts	Dem Districts	Competive Districts	Mean-Median Dist	Votes Bias	Seats Bias	Efficiency Gap	Compactness
Overturned Congress Plan	Congress	14	8	3	3 (2R, 1D)	5.78%	3.68%	16.86%	17.32%	51
Legislature Congress Plan	Congress	14	6	3	5 (2R, 3D)	0.66%	1.27%	5.27%	6.37%	45
Harper et al. Congress Plan	Congress	14	6	4	4 (1R, 3 D)	0.05%	0.32%	0.93%	1.50%	66
LCV et al. Congress Plan	Congress	14	5	3	6 (1R, 5D)	-1.66%	-0.10%	-0.36%	0.67%	74
Overturned Senate Plan	Senate	50	24	17	9 (5R, 4D)	3.66%	3.31%	7.22%	7.14%	61
Legislature Senate Plan	Senate	50	24	17	9 (4R, 5D)	0.77%	2.02%	4.07%	4.24%	69
Harper et al. Senate Plan	Senate	50	21	19	10 (7R, 3D)	-0.08%	0.48%	1.07%	1.21%	63
LCV et al. Senate Plan	Senate	50	22	17	11 (4R, 7D)	-0.07%	0.72%	1.56%	1.67%	69
Overturned House Plan	House	120	56	40	24 (14R, 10D)	3.61%	2.94%	6.77%	6.71%	65
Legislature House Plan	House	120	54	43	23 (9R,14D)	0.89%	1.29%	2.70%	2.72%	72
LCV et al. House Plan	House	120	55	44	21 (7R, 14D)	1.11%	0.91%	1.69%	1.58%	81

Because lack of constitutionality must be established before any consideration can be given to choosing an alternative map, here I will limit myself to political science perspectives on the constitutionality of each of the legislature's proposed maps. I will not discuss the question of which alternative map should be adopted by the court if the map proposed by the legislature is found to be unconstitutional, except to note that the maps proposed by one or more plaintiffs would seem to be ones that the Court could adopt (perhaps as is, perhaps with very minor modifications) if the corresponding legislative map was struck down. However, while I will not discuss which alternative map is best, since that issue is premature, I will use the alternative maps to show how much closer to zero values on the various metrics it would have been possible to come.

My discussion will be limited to the data presented in Table 1, which reports only metrics calculated at the statewide level.[20] I recognize that the information in this table is not the only relevant material. Thus, my conclusions might be changed upon exposure to expert witness testimony about the various plans. In particular, I am not able to incorporate into my conclusions finding about the maps in terms of the spatial configurations of individual districts or county clusters and how those might have been manipulated For these reasons, I have labeled my Report a Preliminary Report.

Before I turn to the three specific maps proposed by the legislature I should note that, on virtually all statistical metrics, the new plans are significant improvements from the old plans. But the plans previously rejected by the Court were such egregious gerrymanders that the standard of doing better is a very low bar. I would also note that perusal of Figure 4 in 22.2.21 NCLV Plaintiffs' Remedial Comments (at p. 18) suggests that the new proposed congressional map has the most pro-Republican bias of the three proposed maps, and the State House map has the least pro-Republican bias. This is generally consistent with my own findings. Thus, a legal decision about which proposed maps are constitutional/unconstitutional need not be the same for all three maps.

Congress

There are several key facts about the congressional map proposed by the legislature.

First and foremost, in a state that is in recent history one that is nearly evenly divided, it creates a distribution of voting strength across districts that is very lopsidedly Republican: 6 Republican leaning districts that, based on averaged recent data will, barring a political tsunami, elect Republicans; 3 Democratic leaning districts that will, barring a political tsunami, elect Democrats; and 5 competitive districts. A sports analogy may be helpful here. Imagine a playoff series of 14 games of which a majority (9 of 14) have already been played, with five games still to go. The team that has won only 3 of the 9 games would need to win all five of the remaining

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[20] I believe the data presented in Table 1 to be a faithful representation of what is found App for the various metrics in Dave's Redistricting, but I recognize that there is always the possibility of error in converting shape files from one GIS program to another and always the possibility of typographical error in my entering data into this Report.

games in order to win the series, and it would need to win four of the five just to get a tie. If the teams were evenly matched in the remaining games of the series the likelihood of winning all five is under 5%.[21] Of course, we need to examine much more closely the expected degree of competition in the districts that DRA labels competitive districts in the proposed congressional map. While there is an apparent Democratic 3-2 advantage in the competitive seats, a close look at the data shows that in 2 of the 3 competitive seats showing a mean Democratic edge that edge is razor thin, and smaller than the still narrow pro-Republican edge in the two Republican leaning competitive districts, while the 3rd district labeled as competitive has a substantial Democratic edge and is a very heavily African-American district Looking at vote margins more closely, we might thus view this map as {6R, 4D, 4 very competitive}.[22] But even so, Democrats would still have to win four of the four competitive seats to win a majority in the delegation.

Second, while the results in the median district look a lot like the statewide average, but with a slight Republican edge, the median is only one district and we must look at the overall map. Here the 5.27% seats bias suggest a substantial pro-Republican bias in terms of the likelihood that a majority of the voters will be able to win a majority of the seats, and the 1.27% vote bias suggests that only a win by more than 50% of the statewide vote can yield the Democrats a majority of the seats. When we compare these levels of partisan bias to the level of partisan bias in the Harper and NCLCV maps we see that each of these two bias measures is multiple times higher in the legislative map than in the alternatives and, even when we look at differences in absolute value rather than ratios, it is still clear that the legislatively proposed congressional map is much more extreme with respect to partisan bias.

Third, the level of compactness of the districts in the previous map was a statistical outlier relative to the ensembles (Chen Expert Report PX482, pp. 17-19) and since the DRA compactness score the new congressional map proposed by the legislature is even lower, my expectation is that, with respect to district compactness the new map will also be a clear statistical outlier. However, unlike its predecessor (Chen Expert Report PX482, Figure 1, p. 14), doing a visual check, the new congressional map does not appear to split any counties in more than two pieces.

Fourth, there has been a substantial drop in the efficiency gap in the new map as compared to the congressional map found to be unconstitutional. But the efficiency gap is not directly a test for

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[21] Of course, this is an improved situation for the Democrats compared to the enacted congressional map, since that map (8 Rep, 3 Dem, 3 competitive) in effect said that the outcome was foreordained before the last three games were played. Barring a political tsunami, that map locked in a permanent Republican majority, and it was shown in the expert witness testimony to make a 10R-4D outcome very likely. Of course, that map was also one of the handful of most blatant and egregious partisan gerrymanders in the nation.

[22] Note that to do this exactly we would need to look election by election to see how often Democrats won, since the mean vote share averaged across elections can lead to misleading conclusions because of variation in Democratic performance. See discussion of essentially this point in Dr. Duchin's Rebuttal Expert Witness Report (PX234).

bias; rather it measures, roughly speaking, how far from a responsiveness level of 2 a map implements. As Dr. Duchin has argued in her previous work, in a view that I share, high values of the efficiency gap are a sign that something may be seriously wrong and signal a need to investigate carefully. However, in my view, low values of the efficiency gap, are not a proof that there is no vote dilution. By offering a map with an efficiency gap of 6.37% for their congressional map, i.e., one with an efficiency gap below 7, the legislative map drawers have apparently sought to draw a congressional map that just narrowly pass a supposed threshold test for partisan gerrymandering (see Memorandum on Remedial Process 4876-1419-931, at p. 7). And the efficiency gap is still a result in a pro-Republican direction.

Because they all point in the same direction, the political effects statistical indicators of partisan gerrymandering strongly suggest the conclusion that this congressional map should be viewed as a pro-Republican gerrymander, but whether these gerrymandering effect rises to the level of a constitutional violation must, of course, be left to legal determination. On the other hand, if I am correct that the compactness of the districts is at a level to show proof of severe outlier status, that in and of itself may be sufficient reason to reject the plan. But of course, that again is entirely a legal question up to the Court to resolve.

NC Senate

My analysis and conclusions for the legislatively proposed NC Senate map are similar to those for legislatively proposed congressional map. In a state that is in recent history one that is nearly evenly divided, this map, too, creates a distribution of voting strength across districts that is very lopsidedly Republican: 24 Republican leaning districts that, based on averaged recent data will, barring a political tsunami, elect Republicans; 17 Democratic leaning districts that will, barring a political tsunami, elect Democrats; and 5 competitive districts. Democrats would have to win nine of the nine competitive seats to win a majority in the Senate.

Second, while the median district again looks a lot like the statewide average, but again with a slight Republican edge, the median is only one district and we must look at the overall map. Here the 4.07% seats bias still suggest a <u>substantial</u> pro-Republican bias in terms of the likelihood that a majority of the voters will be able to win a majority of the seats, even though it is one percentage point or so lower than the comparable statistic in the congressional map, while the 2.00 % vote bias suggests that only a win by considerably more than 50% of the statewide vote can yield the Democrats a majority of the seats. Indeed, on this metric the new NC Senate map is more extreme by nearly a percentage point than the new NC House map. When we compare these levels of partisan bias to the level of partisan bias in the Harper and NCLCV maps we see that each of these two bias measures is at least twice as high in the legislative map as in the alternatives and, even when we look at differences in absolute value rather than ratios, it is still clear that the legislatively proposed congressional map is much more extreme with respect to partisan bias than either of the alternatives.

Third, the compactness level in the Senate map is comparable or higher than that in the alternative Senate maps.

Fourth, there has been a substantial drop in the efficiency gap in the new map as compared to the congressional map found to be unconstitutional. But it remains in a pro-Republican direction.

Because they all point in the same direction, the political effects statistical indicators of partisan gerrymandering argue for the conclusion that this NC Senate map should be viewed as a pro-Republican gerrymander. While, overall, the dilutive effects of this map do not appear quite as severe as in the congressional map they are still still quite substantial. However, I have not had time to analyze how the map may have been manipulated at the level of individual districts in terms of things like city cuts or county transversals. Of course, whether the clear indicators of partisan gerrymandering effects identified in Table 1 and my discussion rise to the level of a constitutional violation requires determination by this Court.

NC State House

My analysis for the legislatively proposed NC House map uses the same approach as for the previously considered maps. In a state that is in recent history one that is nearly evenly divided, this map, too, creates a distribution of voting strength across districts that is very lopsidedly Republican: 54 Republican leaning districts that, based on averaged recent data will, barring a political tsunami, elect Republicans; 43 Democratic leaning districts that will, barring a political tsunami, elect Democrats; and 23 competitive districts. In the House, however, unlike the other maps, the Democrats do not have to win all of the competitive seats to win a majority in the House. Moreover, unlike the other two proposed maps, when we look at the proposed NC House map we see that the competitive seats are substantially Democrat in directionality (9R, 14D). This map is genuinely far more competitive than either of the other two legislatively proposed maps even though (see below) it remains tilted in a pro-Republican direction.

Second, while the median district again looks a lot like the statewide average, but again with a slight Republican edge, the 2.70% seats bias still suggest a <u>substantial</u> pro-Republican bias in terms of the likelihood that a majority of the voters will be able to win a majority of the seats. But the value on this metric is one which is more than one percentage point lower than the comparable statistic in the Senate map, and the 1.29% vote bias in this map is again almost one percentage point lower than the 2.00 value of this metric for the Senate. But arguably quit important in judging the constitutionality of this map in the full context are the facts that: (a) the Harper plaintiffs have not chosen to offer an alternative NC House map but are apparently content to see the legislative map implemented by the Court, (b) the map was passed by a clear bipartisan consensus in the legislature, including members of the legislature who belong to particular minority communities, and (c) that while it still is further from being non-dilutive than the NCLCV House map alternative, it is <u>far</u> closer to Plaintiffs' map than it is to the rejected enacted NC House map.

Third, the compactness level in the Senate map is high relative to the other maps in Table 1, even though the NCLCV House map alternative has an even higher score.

Fourth, there has been a substantial drop in the efficiency gap in the new map as compared to the NC House map found to be unconstitutional. It is at the low level of 2.72 even though it remains in a pro-Republican direction.

I have not had time to analyze how this map may have been manipulated at the level of individual districts in terms of things like city cuts or county transversals or racial fragmentaion. But of the three legislatively proposed maps, for the reasons given above, this is the one that I would feel most comfortable with seeing ordered by the Court. Looking at the totality of the circumstances insofar as these are presently known to me, and recognizing that this map is still not ideal (nor need it be), this legislatively proposed NC House map simply lacks the same clear indicia of egregious bias found in the previously rejected maps and still found, but to a lesser extent than in the rejected maps, in the legislatively proposed maps for Congress and for the NC Senate that I discuss above.

```
> summary(towns_sims_assemmb)
SMC: 10,000 sampled plans of 90 districts on 7,305 units
`adapt_k_thresh`=0.985 • `seq_alpha`=0.5
`est_label_mult`=1 • `pop_temper`=0

Plan diversity 80% range: 0.077 to 0.812
✖ WARNING: Low plan diversity

Sampling diagnostics for SMC run 1 of 1 (10,000 samples)
      Eff. samples (%) Acc. rate Log wgt. sd  Max. unique Est. k
Split 1    8,872 (88.7%)    37.0%      0.59 6,333 (100%)    16
Split 2    8,735 (87.4%)    36.5%      0.66 6,133 ( 97%)    16
Split 3    8,767 (87.7%)    51.8%      0.65 6,041 ( 96%)    11
Split 4    8,761 (87.6%)    38.1%      0.64 6,151 ( 97%)    15
Split 5    8,781 (87.8%)    50.8%      0.63 6,160 ( 97%)    11
Split 6    8,815 (88.1%)    54.1%      0.63 6,058 ( 96%)    10
Split 7    8,804 (88.0%)    45.1%      0.64 6,143 ( 97%)    12
Split 8    8,797 (88.0%)    48.1%      0.65 6,104 ( 97%)    11
Split 9    8,828 (88.3%)    47.3%      0.66 6,114 ( 97%)    11
Split 10   8,754 (87.5%)    39.4%      0.70 6,100 ( 97%)    13
Split 11   8,714 (87.1%)    61.0%      0.72 6,086 ( 96%)     8
Split 12   8,718 (87.2%)    81.2%      0.74 6,054 ( 96%)     5
Split 13   8,761 (87.6%)    48.8%      0.76 6,113 ( 97%)    10
Split 14   8,725 (87.3%)    59.0%      0.78 6,077 ( 96%)     8
Split 15   8,652 (86.5%)    48.1%      0.81 6,060 ( 96%)    10
Split 16   8,649 (86.5%)    64.1%      0.84 6,075 ( 96%)     7
Split 17   8,700 (87.0%)    70.3%      0.84 6,100 ( 97%)     6
Split 18   8,528 (85.3%)    55.6%      0.87 6,058 ( 96%)     8
Split 19   8,483 (84.8%)    45.3%      0.90 6,121 ( 97%)    10
```

Split 20	8,439 (84.4%)	67.9%	0.92	6,068 (96%)	6
Split 21	8,381 (83.8%)	53.9%	0.95	6,011 (95%)	8
Split 22	8,478 (84.8%)	35.6%	0.96	5,988 (95%)	12
Split 23	8,347 (83.5%)	51.9%	0.99	6,024 (95%)	8
Split 24	8,287 (82.9%)	65.1%	1.02	6,091 (96%)	6
Split 25	7,946 (79.5%)	50.9%	1.06	6,008 (95%)	8
Split 26	8,065 (80.7%)	37.1%	1.08	5,935 (94%)	11
Split 27	8,129 (81.3%)	55.2%	1.09	6,048 (96%)	7
Split 28	8,247 (82.5%)	55.0%	1.12	6,040 (96%)	7
Split 29	8,244 (82.4%)	60.9%	1.17	6,041 (96%)	6
Split 30	8,266 (82.7%)	52.8%	1.23	6,014 (95%)	7
Split 31	8,098 (81.0%)	60.0%	1.27	5,986 (95%)	6
Split 32	8,099 (81.0%)	51.3%	1.31	6,012 (95%)	7
Split 33	8,125 (81.3%)	65.7%	1.35	5,996 (95%)	5
Split 34	8,228 (82.3%)	72.6%	1.37	6,056 (96%)	4
Split 35	8,198 (82.0%)	55.2%	1.43	5,999 (95%)	6
Split 36	8,172 (81.7%)	71.5%	1.48	6,065 (96%)	4
Split 37	8,102 (81.0%)	61.2%	1.55	6,042 (96%)	5
Split 38	8,116 (81.2%)	52.9%	1.59	5,960 (94%)	6
Split 39	8,141 (81.4%)	68.3%	1.62	5,922 (94%)	4
Split 40	8,085 (80.9%)	77.5%	1.67	5,983 (95%)	3
Split 41	8,042 (80.4%)	67.0%	1.72	5,986 (95%)	4
Split 42	8,000 (80.0%)	75.8%	1.76	5,929 (94%)	3
Split 43	7,905 (79.1%)	75.1%	1.77	5,995 (95%)	3
Split 44	7,907 (79.1%)	73.9%	1.82	5,960 (94%)	3
Split 45	7,675 (76.8%)	62.3%	1.85	5,974 (95%)	4
Split 46	7,830 (78.3%)	71.1%	1.89	5,917 (94%)	3
Split 47	7,721 (77.2%)	70.6%	1.98	6,019 (95%)	3
Split 48	7,560 (75.6%)	58.8%	2.11	5,854 (93%)	4

Split 49	7,485 (74.9%)	68.6%	2.16	5,896 (93%)	3
Split 50	7,489 (74.9%)	55.8%	2.12	5,863 (93%)	4
Split 51	7,378 (73.8%)	66.1%	2.15	5,865 (93%)	3
Split 52	7,384 (73.8%)	33.8%	2.13	5,833 (92%)	7
Split 53	7,281 (72.8%)	45.1%	2.13	5,808 (92%)	5
Split 54	7,139 (71.4%)	63.8%	2.12	5,830 (92%)	3
Split 55	7,041 (70.4%)	51.1%	2.12	5,726 (91%)	4
Split 56	7,041 (70.4%)	36.0%	2.12	5,686 (90%)	6
Split 57	6,862 (68.6%)	49.1%	2.14	5,779 (91%)	4
Split 58	6,548 (65.5%)	40.4%	2.15	5,703 (90%)	5
Split 59	6,564 (65.6%)	58.5%	2.21	5,720 (90%)	3
Split 60	6,561 (65.6%)	55.4%	2.22	5,656 (89%)	3
Split 61	6,400 (64.0%)	37.1%	2.27	5,688 (90%)	5
Split 62	6,241 (62.4%)	43.9%	2.27	5,646 (89%)	4
Split 63	6,347 (63.5%)	34.9%	2.32	5,562 (88%)	5
Split 64	6,200 (62.0%)	40.3%	2.35	5,600 (89%)	4
Split 65	5,693 (56.9%)	39.6%	2.38	5,519 (87%)	4
Split 66	5,246 (52.5%)	31.5%	2.43	5,468 (87%)	5
Split 67	3,587 (35.9%)	37.6%	2.57	5,412 (86%)	4
Split 68	3,220 (32.2%)	46.4%	2.71	4,974 (79%)	3
Split 69	4,526 (45.3%)	29.2%	2.32	4,641 (73%)	5
Split 70	5,195 (51.9%)	27.6%	1.79	5,034 (80%)	5
Split 71	5,400 (54.0%)	31.4%	1.61	5,291 (84%)	4
Split 72	5,521 (55.2%)	37.5%	1.54	5,348 (85%)	3
Split 73	4,993 (49.9%)	46.2%	1.53	5,336 (84%)	2
Split 74	5,417 (54.2%)	32.5%	1.52	5,271 (83%)	3
Split 75	5,265 (52.7%)	29.9%	1.56	5,296 (84%)	3
Split 76	5,078 (50.8%)	21.8%	1.64	5,347 (85%)	4
Split 77	4,605 (46.0%)	27.1%	1.76	5,239 (83%)	3

Split 78	4,569 (45.7%)	18.8%	1.98	5,110 (81%)	4
Split 79	4,688 (46.9%)	21.8%	2.19	5,019 (79%)	3
Split 80	3,968 (39.7%)	19.6%	2.35	4,882 (77%)	3
Split 81	4,113 (41.1%)	13.7%	2.42	4,742 (75%)	4
Split 82	3,936 (39.4%)	13.0%	2.49	4,701 (74%)	4
Split 83	4,093 (40.9%)	14.1%	2.46	4,650 (74%)	3
Split 84	3,667 (36.7%)	12.1%	2.40	4,555 (72%)	3
Split 85	3,988 (39.9%)	8.1%	2.31	4,525 (72%)	4
Split 86	3,648 (36.5%)	9.4%	2.24	4,483 (71%)	3
Split 87	3,995 (39.9%)	5.6%	2.13	4,333 (69%)	5
Split 88	4,236 (42.4%)	7.6%	1.88	4,429 (70%)	3
Split 89	4,795 (48.0%)	3.4%	1.73	4,112 (65%)	2
Resample	957 (9.6%)	NA%	1.80	3,865 (61%)	NA

• Watch out for low effective samples, very low acceptance rates (less than 1%), large std. devs. of the log weights (more than 3 or so), and low numbers of unique plans. R-hat values for summary statistics should be between 1 and 1.05.

• Low diversity: Check for potential bottlenecks. Increase the number of samples. Examine the diversity plot with `hist(plans_diversity(towns_sims_assemmb), breaks=24)`. Consider weakening or removing constraints, or increasing the population tolerance. If the acceptance rate drops quickly in the final splits, try increasing `pop_temper` by 0.01.

--

> summary(towns_sims_senate)

SMC: 10,000 sampled plans of 30 districts on 7,305 units

`adapt_k_thresh`=0.985 • `seq_alpha`=0.5

`est_label_mult`=1 • `pop_temper`=0

Plan diversity 80% range: 0.58 to 0.78

Sampling diagnostics for SMC run 1 of 1 (10,000 samples)

	Eff. samples (%)	Acc. rate	Log wgt. sd	Max. unique	Est. k
Split 1	9,477 (94.8%)	26.2%	0.45	6,319 (100%)	17
Split 2	9,354 (93.5%)	38.1%	0.51	6,223 (98%)	11
Split 3	9,316 (93.2%)	54.7%	0.54	6,220 (98%)	7
Split 4	9,238 (92.4%)	54.0%	0.56	6,236 (99%)	7
Split 5	9,197 (92.0%)	38.1%	0.57	6,223 (98%)	10
Split 6	9,122 (91.2%)	56.1%	0.57	6,127 (97%)	6
Split 7	9,051 (90.5%)	53.9%	0.58	6,136 (97%)	6
Split 8	8,977 (89.8%)	66.5%	0.60	6,129 (97%)	4
Split 9	8,932 (89.3%)	56.1%	0.61	6,085 (96%)	5
Split 10	8,754 (87.5%)	42.1%	0.64	6,143 (97%)	7
Split 11	8,628 (86.3%)	51.5%	0.68	6,150 (97%)	5
Split 12	8,517 (85.2%)	44.1%	0.72	6,100 (97%)	6
Split 13	8,506 (85.1%)	54.8%	0.76	6,076 (96%)	4
Split 14	8,479 (84.8%)	39.9%	0.80	6,061 (96%)	6
Split 15	8,376 (83.8%)	23.6%	0.84	6,013 (95%)	10
Split 16	8,271 (82.7%)	31.5%	0.89	6,007 (95%)	7
Split 17	8,244 (82.4%)	39.6%	0.93	5,969 (94%)	5
Split 18	8,125 (81.3%)	37.4%	0.97	5,937 (94%)	5
Split 19	7,817 (78.2%)	41.3%	1.02	5,956 (94%)	4
Split 20	7,834 (78.3%)	23.8%	1.05	5,875 (93%)	7
Split 21	7,781 (77.8%)	30.1%	1.10	5,917 (94%)	5
Split 22	7,709 (77.1%)	20.3%	1.16	5,820 (92%)	7
Split 23	7,681 (76.8%)	25.1%	1.22	5,740 (91%)	5
Split 24	7,566 (75.7%)	27.3%	1.27	5,726 (91%)	4

Split 25	7,238 (72.4%)	29.8%	1.30	5,656 (89%)	3
Split 26	7,551 (75.5%)	17.2%	1.28	5,553 (88%)	5
Split 27	7,614 (76.1%)	17.3%	1.24	5,436 (86%)	4
Split 28	7,245 (72.4%)	17.4%	1.20	5,258 (83%)	3
Split 29	7,957 (79.6%)	8.6%	1.05	4,704 (74%)	2
Resample	3,514 (35.1%)	NA%	0.91	6,404 (101%)	NA

• Watch out for low effective samples, very low acceptance rates (less than 1%), large std. devs. of the log weights (more than 3 or so), and low numbers of unique plans. R-hat values for summary statistics should be between 1 and 1.05.

```
> summary(state_plans)

SMC: 20,000 sampled plans of 99 districts on 7,059 units

`adapt_k_thresh`=0.985 • `seq_alpha`=0.5

`est_label_mult`=1 • `pop_temper`=0


Plan diversity 80% range: 0.048 to 0.944

✕ WARNING: Low plan diversity


Sampling diagnostics for SMC run 1 of 1 (20,000 samples)

         Eff. samples (%) Acc. rate Log wgt. sd  Max. unique Est. k

Split 1    9,717 (48.6%)    35.4%        2.8 12,725 (101%)    15

Split 2    8,719 (43.6%)    57.8%        2.9  9,629 ( 76%)     9

Split 3    8,356 (41.8%)    51.8%        2.9  9,198 ( 73%)    10

Split 4    8,328 (41.6%)    62.2%        2.9  9,219 ( 73%)     8

Split 5    8,163 (40.8%)    62.2%        2.9  9,049 ( 72%)     8

Split 6    7,994 (40.0%)    50.3%        2.9  9,101 ( 72%)    10

Split 7    7,760 (38.8%)    54.6%        2.9  9,054 ( 72%)     9

Split 8    7,685 (38.4%)    54.0%        3.0  8,903 ( 70%)     9

Split 9    7,639 (38.2%)    73.2%        3.0  8,884 ( 70%)     6

Split 10   7,437 (37.2%)    59.3%        2.9  8,848 ( 70%)     8

Split 11   7,239 (36.2%)    52.5%        3.0  8,860 ( 70%)     9

Split 12   7,124 (35.6%)    71.8%        3.0  8,819 ( 70%)     6

Split 13   6,974 (34.9%)    78.8%        2.9  8,723 ( 69%)     5

Split 14   6,662 (33.3%)    78.0%        3.0  8,706 ( 69%)     5

Split 15   6,676 (33.4%)    63.3%        3.0  8,725 ( 69%)     7

Split 16   6,630 (33.1%)    69.6%        3.0  8,615 ( 68%)     6

Split 17   6,328 (31.6%)    77.3%        3.0  8,575 ( 68%)     5

Split 18   6,318 (31.6%)    45.1%        3.0  8,415 ( 67%)    10

Split 19   5,939 (29.7%)    60.9%        3.0  8,386 ( 66%)     7
```

Split 20	5,909 (29.5%)	60.5%	3.0	8,337 (66%)	7
Split 21	5,469 (27.3%)	53.6%	3.0	8,337 (66%)	8
Split 22	5,582 (27.9%)	59.9%	3.0	8,281 (66%)	7
Split 23	6,003 (30.0%)	53.4%	3.0	8,154 (64%)	8
Split 24	5,680 (28.4%)	74.6%	3.0	8,335 (66%)	5
Split 25	5,672 (28.4%)	59.5%	3.0	8,220 (65%)	7
Split 26	5,687 (28.4%)	73.7%	3.0	8,284 (66%)	5
Split 27	5,302 (26.5%)	42.4%	3.1	8,182 (65%)	10
Split 28	5,217 (26.1%)	64.9%	3.1	8,169 (65%)	6
Split 29	4,759 (23.8%)	41.4%	3.1	8,171 (65%)	10
Split 30	4,178 (20.9%)	63.9%	3.1	8,092 (64%)	6
Split 31	4,272 (21.4%)	45.2%	3.1	8,101 (64%)	9
Split 32	4,162 (20.8%)	44.2%	3.2	7,838 (62%)	9
Split 33	3,478 (17.4%)	62.1%	3.2	7,848 (62%)	6
Split 34	3,461 (17.3%)	70.2%	3.2	7,765 (61%)	5
Split 35	2,891 (14.5%)	43.2%	3.2	7,771 (61%)	9
Split 36	3,543 (17.7%)	53.7%	3.2	7,638 (60%)	7
Split 37	3,979 (19.9%)	53.3%	3.1	7,633 (60%)	7
Split 38	4,170 (20.8%)	41.6%	3.0	7,793 (62%)	9
Split 39	4,488 (22.4%)	59.2%	2.9	7,951 (63%)	6
Split 40	4,932 (24.7%)	58.5%	2.8	8,170 (65%)	6
Split 41	5,427 (27.1%)	57.6%	2.8	8,361 (66%)	6
Split 42	5,326 (26.6%)	73.7%	2.8	8,532 (67%)	4
Split 43	5,498 (27.5%)	73.5%	2.8	8,620 (68%)	4
Split 44	5,310 (26.5%)	38.4%	2.8	8,662 (69%)	9
Split 45	5,603 (28.0%)	55.1%	2.8	8,518 (67%)	6
Split 46	5,798 (29.0%)	47.6%	2.9	8,751 (69%)	7
Split 47	5,831 (29.2%)	61.2%	2.9	8,705 (69%)	5
Split 48	5,810 (29.1%)	69.4%	2.9	8,820 (70%)	4

Split 49	6,098 (30.5%)	40.4%	2.8	8,713 (69%)	8
Split 50	6,198 (31.0%)	59.2%	2.8	8,846 (70%)	5
Split 51	6,607 (33.0%)	51.1%	2.8	8,865 (70%)	6
Split 52	6,697 (33.5%)	49.9%	2.7	8,864 (70%)	6
Split 53	6,534 (32.7%)	65.8%	2.8	8,949 (71%)	4
Split 54	6,584 (32.9%)	55.8%	2.8	8,938 (71%)	5
Split 55	7,005 (35.0%)	36.3%	2.7	9,004 (71%)	8
Split 56	6,741 (33.7%)	54.5%	2.8	9,045 (72%)	5
Split 57	6,773 (33.9%)	72.0%	2.8	8,995 (71%)	3
Split 58	6,446 (32.2%)	52.3%	2.8	9,022 (71%)	5
Split 59	6,202 (31.0%)	38.5%	2.8	8,964 (71%)	7
Split 60	6,302 (31.5%)	37.9%	2.9	8,820 (70%)	7
Split 61	6,198 (31.0%)	43.0%	2.9	8,884 (70%)	6
Split 62	6,022 (30.1%)	58.6%	3.0	8,718 (69%)	4
Split 63	5,706 (28.5%)	48.4%	3.0	8,596 (68%)	5
Split 64	5,461 (27.3%)	55.5%	3.1	8,550 (68%)	4
Split 65	5,360 (26.8%)	65.8%	3.1	8,470 (67%)	3
Split 66	5,071 (25.4%)	45.9%	3.1	8,467 (67%)	5
Split 67	5,285 (26.4%)	45.1%	3.1	8,357 (66%)	5
Split 68	4,976 (24.9%)	43.9%	3.1	8,295 (66%)	5
Split 69	4,762 (23.8%)	36.5%	3.0	8,331 (66%)	6
Split 70	4,670 (23.4%)	49.4%	3.0	8,259 (65%)	4
Split 71	4,464 (22.3%)	48.4%	3.0	8,262 (65%)	4
Split 72	4,569 (22.8%)	58.1%	3.0	8,069 (64%)	3
Split 73	3,646 (18.2%)	39.0%	2.8	8,170 (65%)	5
Split 74	4,417 (22.1%)	45.7%	2.8	8,100 (64%)	4
Split 75	2,855 (14.3%)	46.0%	2.8	8,143 (64%)	4
Split 76	4,843 (24.2%)	54.0%	2.7	8,031 (64%)	3
Split 77	5,135 (25.7%)	41.8%	2.6	8,286 (66%)	4

Split 78	4,060 (20.3%)	40.6%	2.6	8,724 (69%)	4	
Split 79	3,686 (18.4%)	27.1%	2.7	8,593 (68%)	6	
Split 80	4,160 (20.8%)	38.0%	2.7	8,232 (65%)	4	
Split 81	4,236 (21.2%)	36.4%	2.6	8,169 (65%)	4	
Split 82	4,535 (22.7%)	43.4%	2.6	8,297 (66%)	3	
Split 83	5,243 (26.2%)	52.8%	2.5	8,431 (67%)	2	
Split 84	5,122 (25.6%)	25.1%	2.4	8,724 (69%)	5	
Split 85	5,328 (26.6%)	14.9%	2.4	8,811 (70%)	8	
Split 86	5,477 (27.4%)	22.8%	2.4	8,831 (70%)	5	
Split 87	5,472 (27.4%)	17.9%	2.3	8,843 (70%)	6	
Split 88	5,714 (28.6%)	25.0%	2.3	8,931 (71%)	4	
Split 89	5,186 (25.9%)	29.9%	2.2	8,961 (71%)	3	
Split 90	4,849 (24.2%)	35.9%	2.2	8,917 (71%)	2	
Split 91	5,560 (27.8%)	14.3%	2.2	8,736 (69%)	5	
Split 92	5,225 (26.1%)	15.1%	2.2	8,688 (69%)	4	
Split 93	4,263 (21.3%)	17.3%	2.2	8,339 (66%)	3	
Split 94	4,078 (20.4%)	14.8%	2.3	7,968 (63%)	3	
Split 95	5,491 (27.5%)	15.9%	2.5	7,812 (62%)	2	
Split 96	5,517 (27.6%)	8.7%	2.6	7,816 (62%)	3	
Split 97	4,108 (20.5%)	10.3%	2.8	7,346 (58%)	2	
Split 98	4,712 (23.6%)	4.0%	2.4	7,048 (56%)	2	
Resample	136 (0.7%)	NA%	2.1	4,518 (36%)	NA	*

• Watch out for low effective samples, very low acceptance rates (less than 1%), large std. devs. of the log weights (more than 3 or so), and low numbers of unique plans. R-hat values for summary statistics should be between 1 and 1.05.

• Low diversity: Check for potential bottlenecks. Increase the number of samples. Examine the diversity plot with `hist(plans_diversity(state_plans), breaks=24)`. Consider weakening or removing constraints, or increasing the population tolerance. If the acceptance rate drops quickly in the final splits, try increasing `pop_temper` by 0.01.

• (*) Bottlenecks found: Consider weakening or removing constraints, or increasing the population tolerance. If the acceptance rate drops quickly in the final splits, try increasing `pop_temper` by 0.01. If the weight variance (Log wgt. sd) increases steadily or is particularly large for the "Resample" step, consider increasing `seq_alpha`. To visualize what geographic areas may be causing problems, try running the following code. Highlighted areas are those that may be causing the bottleneck.

```
plot(<map object>, rowMeans(as.matrix(state_plans) == <bottleneck iteration>))
```

SMC: 20,000 sampled plans of 33 districts on 7,059 units

`adapt_k_thresh`=0.985 • `seq_alpha`=0.5

`est_label_mult`=1 • `pop_temper`=0

Plan diversity 80% range: 0.20 to 0.98

✕ WARNING: Low plan diversity

Sampling diagnostics for SMC run 1 of 1 (20,000 samples)

	Eff. samples (%)	Acc. rate	Log wgt. sd	Max. unique	Est. k
Split 1	2,471 (12.4%)	22.3%	7.4	12,609 (100%)	16
Split 2	2,182 (10.9%)	34.7%	7.4	4,665 (37%)	10
Split 3	2,004 (10.0%)	47.5%	7.3	4,543 (36%)	7
Split 4	1,846 (9.2%)	47.0%	7.3	4,439 (35%)	7
Split 5	1,789 (8.9%)	46.4%	7.2	4,309 (34%)	7
Split 6	1,676 (8.4%)	58.3%	7.2	4,263 (34%)	5
Split 7	1,654 (8.3%)	57.8%	7.1	4,272 (34%)	5
Split 8	1,213 (6.1%)	45.2%	7.1	4,318 (34%)	7

Split 9	1,395 (7.0%)	40.2%	7.1	4,036 (32%)	8	
Split 10	683 (3.4%)	56.1%	6.9	4,351 (34%)	5	*
Split 11	829 (4.1%)	63.3%	6.7	4,525 (36%)	4	*
Split 12	1,684 (8.4%)	54.8%	6.6	5,492 (43%)	5	
Split 13	2,093 (10.5%)	37.1%	6.5	6,209 (49%)	8	
Split 14	2,881 (14.4%)	52.3%	6.5	6,553 (52%)	5	
Split 15	3,491 (17.5%)	39.3%	6.6	6,663 (53%)	7	
Split 16	3,356 (16.8%)	42.9%	6.6	6,863 (54%)	6	
Split 17	3,462 (17.3%)	40.7%	6.6	6,935 (55%)	6	
Split 18	2,710 (13.6%)	52.8%	6.5	7,015 (55%)	4	
Split 19	3,770 (18.9%)	58.8%	6.4	7,018 (56%)	3	
Split 20	3,568 (17.8%)	57.0%	6.3	7,097 (56%)	3	
Split 21	3,370 (16.8%)	40.5%	6.4	6,997 (55%)	5	
Split 22	2,306 (11.5%)	29.0%	6.4	6,805 (54%)	7	
Split 23	2,013 (10.1%)	36.4%	6.5	6,409 (51%)	5	
Split 24	2,601 (13.0%)	25.6%	6.2	5,782 (46%)	7	
Split 25	1,963 (9.8%)	20.2%	5.9	5,851 (46%)	8	
Split 26	1,351 (6.8%)	27.7%	5.1	5,599 (44%)	5	
Split 27	1,338 (6.7%)	36.5%	4.8	5,119 (40%)	3	
Split 28	1,845 (9.2%)	28.9%	4.3	4,818 (38%)	4	
Split 29	1,258 (6.3%)	30.1%	3.8	5,168 (41%)	3	
Split 30	2,319 (11.6%)	20.2%	2.9	5,032 (40%)	4	
Split 31	2,536 (12.7%)	17.0%	3.4	7,313 (58%)	4	
Split 32	5,302 (26.5%)	4.1%	2.1	7,023 (56%)	5	
Resample	90 (0.4%)	NA%	1.9	4,973 (39%)	NA	*

- Watch out for low effective samples, very low acceptance rates (less than 1%), large std. devs. of the log weights (more than 3 or so), and low numbers of unique plans. R-hat values for summary statistics should be between 1 and 1.05.

• Low diversity: Check for potential bottlenecks. Increase the number of samples. Examine the diversity plot with `hist(plans_diversity(state_plans), breaks=24)`. Consider weakening or removing constraints, or increasing the population tolerance. If the acceptance rate drops quickly in the final splits, try increasing `pop_temper` by 0.01.

• (*) Bottlenecks found: Consider weakening or removing constraints, or increasing the population tolerance. If the acceptance rate drops quickly in the final splits, try increasing `pop_temper` by 0.01. If the weight variance (Log wgt. sd) increases steadily or is particularly large for the "Resample" step, consider increasing `seq_alpha`. To visualize what geographic areas may be causing

problems, try running the following code. Highlighted areas are those that may be causing the bottleneck.

```
plot(<map object>, rowMeans(as.matrix(state_plans) == <bottleneck iteration>))
```

FROM THE redist manual: https://alarm-redist.org/redist/reference/summary.redist_plans.html

Details

For SMC and MCMC, if there are multiple runs/chains, R-hat values will be computed for each summary statistic. These values should be close to 1. If they are not, then there is too much between-chain variation, indicating that there are not enough samples. R-hat values are calculated after rank-normalization and folding. MCMC chains are split in half before R-hat is computed. For summary statistics that vary across districts, R-hat is calculated for the first district only.

For SMC, diagnostics statistics include:

Effective samples: the effective sample size at each iteration, computed using the SMC weights. Larger is better. The percentage in parentheses is the ratio of the effective samples to the total samples.

Acceptance rate: the fraction of drawn spanning trees which yield a valid redistricting plan within the population tolerance. Very small values (< 1%) can indicate a bottleneck and may lead to a lack of diversity.

Standard deviation of the log weights: More variable weights (larger s.d.) indicate less efficient sampling. Values greater than 3 are likely problematic.

Maximum unique plans: an upper bound on the number of unique redistricting plans that survive each stage. The percentage in parentheses is the ratio of this number to the total number of samples. Small values (< 100) indicate a bottleneck, which leads to a loss of sample diversity and a higher variance.

Estimated k parameter: How many spanning tree edges were considered for cutting at each split. Mostly informational, though large jumps may indicate a need to increase adapt_k_thresh.

Bottleneck: An asterisk will appear in the right column if a bottleneck appears likely, based on the values of the other statistics.

In the event of problematic diagnostics, the function will provide suggestions for improvement.

Introduction to redist

Source: `vignettes/redist.Rmd` (https://github.com/alarm-redist/redist/blob/HEAD/vignettes/redist.Rmd)

The `redist` package provides algorithms and tools for scalable and replicable redistricting analyses. This vignette introduces the package by way of an analysis of redistricting in the state of Iowa, which can broken down into four distinct steps:

1. Compiling, cleaning, and preparing the data
2. Defining the redistricting problem
3. Simulating redistricting plans
4. Analyzing the simulated plans

First, however, a brief overview of the package itself.

```
library(redist)
library(dplyr)
library(ggplot2)
```

The **`redist`** package

To install `redist`, follow the instructions in the README (../index.html#installation-instructions).

For more information on package components, check out the full documentation (../reference/index.html).

Algorithms

The package contains a variety of redistricting simulation and enumeration algorithms. Generally you will use one of the following three algorithms:

- `redist_smc()`, the recommended algorithm for most purposes.[1]
- `redist_mergesplit()`, a MCMC version of the SMC proposal.[2]
- `redist_flip()`, another MCMC algorithm which uses more local proposals.[3]

The other algorithms are

- `redist.enumpart()` for efficient enumeration of small maps.[4]
- `redist_shortburst()` for optimizing a plan according to a user-provided criterion.[5]
- `redist.rsg()` and `redist.crsg()`, which do not sample from a known target distribution.[6]

Data

The package comes with several built-in datasets, which may be useful in exploring the package's functionality and in becoming familiar with algorithmic redistricting:

- `iowa` (../reference/iowa.html) (used in this vignette).
- `fl25` (../reference/fl25.html), a 25-precinct subset of the state of Florida.
- `fl25_enum` (../reference/fl25_enum.html), containing all possible sets of three districts drawn on the 25-precinct Florida map.
- `fl70` (../reference/fl70.html), a 70-precinct subset of the state of Florida.
- `fl250` (../reference/fl250.html), a 250-precinct subset of the state of Florida.

Compiling, cleaning, and preparing the data

The most time-consuming part of a redistricting analysis, like most data analyses, is collecting and cleaning the necessary data. For redistricting, this data includes geographic shapefiles for precincts and existing legislative district plans, precinct- or block-level demographic information from the Census, and precinct-level political data. These data generally come from different sources, and may not fully overlap with each other, further complicating the problem.

`redist` is not focused on this data collection process. The `geomander` (https://christophertkenny.com/geomander/) package contains many helpful functions for compiling these data, and fixing problems in geographic data.

The ALARM project provides pre-cleaned redistricting data files (https://alarm-redist.org/posts/2021-08-10-census-2020/) consisting of VEST election data joined 2020 Census data at the precinct level. Other sources for precinct-level geographic and political information include the MIT Election Lab (https://electionlab.mit.edu/data), the Census (https://www.census.gov/programs-surveys/decennial-census/about/rdo.html), the Redistricting Data Hub (https://redistrictingdatahub.org/), the Voting and Election Science Team (https://dataverse.harvard.edu/dataverse/electionscience), the Harvard Election Data Archive (https://dataverse.harvard.edu/dataverse/eda), the Metric Geometry and Gerrymandering Group (https://github.com/mggg-states), and some state websites.

Iowa

For this analysis of Iowa, we'll use the data included in the package, which was compiled from the Census and the Harvard Election Data Archive. It contains, for each county, the total population and voting-age population by race, as well as the number of votes for President in 2008. The `geometry` column contains the geographic shapefile information (https://r-spatial.github.io/sf/).

```
data(iowa)
print(iowa)
#> Simple feature collection with 99 features and 15 fields
#> Geometry type: MULTIPOLYGON
#> Dimension:     XY
#> Bounding box:  xmin: 4081849 ymin: 2879102 xmax: 5834228 ymax: 4024957
#> Projected CRS: NAD83(HARN) / Iowa North (ftUS)
#> First 10 features:
#>     fips       name cd_2010    pop  white black hisp    vap  wvap bvap hvap
#> 1  19001      Adair       3   7682   7507    11  101   5957  5860    5   53
#> 2  19003      Adams       3   4029   3922     8   37   3180  3109    6   22
#> 3  19005  Allamakee       1  14330  13325   109  757  11020 10430   82  425
#> 4  19007  Appanoose       2  12887  12470    55  181   9993  9745   40   99
#> 5  19009    Audubon       4   6119   6007     9   37   4780  4714    5   27
#> 6  19011     Benton       1  26076  25387    93  275  19430 19068   49  155
#> 7  19013 Black Hawk       1 131090 109968 11493 4907 102594 89541 7677 2865
#> 8  19015      Boone       4  26306  25194   202  505  20027 19448  103  260
#> 9  19017     Bremer       1  24276  23459   186  239  18763 18242  155  137
#> 10 19019   Buchanan       1  20958  20344    59  243  15282 14979   32  128
#>    tot_08 dem_08 rep_08    region                       geometry
#> 1    4053   1924   2060     South MULTIPOLYGON (((4592338 328...
#> 2    2206   1118   1046     South MULTIPOLYGON (((4528041 315...
#> 3    7059   3971   2965 Northeast MULTIPOLYGON (((5422507 401...
#> 4    6176   2970   3086     South MULTIPOLYGON (((5032545 306...
#> 5    3435   1739   1634 Northwest MULTIPOLYGON (((4487363 341...
#> 6   13712   7058   6447 Southeast MULTIPOLYGON (((5246216 357...
#> 7   64775  39184  24662 Northeast MULTIPOLYGON (((5175640 369...
#> 8   13929   7356   6293   Central MULTIPOLYGON (((4741174 354...
#> 9   12871   6940   5741 Northeast MULTIPOLYGON (((5174636 379...
#> 10  10338   6050   4139 Northeast MULTIPOLYGON (((5302846 370...
```

Defining the redistricting problem

A redistricting problem is defined by the map of the precincts, the number of contiguous districts to divide the precincts into, the level of population parity to enforce, and any other necessary constraints that must be imposed.

Determining the relevant constraints

In Iowa, congressional districts are constructed not out of precincts but out of the state's 99 counties, and in the 2010 redistricting cycle, Iowa was apportioned four congressional districts, down one from the 2000 cycle. Chapter 42 of the Iowa Code provides guidance on the other constraints imposed on the redistricting process (our emphasis added):

42.4 Redistricting standards.

…

1.b. Congressional districts shall each have a population *as nearly equal as practicable* to the ideal district population, derived as prescribed in paragraph "a" of this subsection. No congressional district shall have a population which varies by more than *one percent from the applicable ideal district population*, except as necessary to comply with Article III, section 37 of the Constitution of the State of Iowa.

…

3. Districts shall be composed of convenient *contiguous territory*. Areas which meet only at the points of adjoining corners are not contiguous.

4. Districts shall be *reasonably compact* in form, to the extent consistent with the standards established by subsections 1, 2, and 3. In general, reasonably compact districts are those which are square, rectangular, or hexagonal in shape, and not irregularly shaped, to the extent permitted by natural or political boundaries….

5. No district shall be drawn for the purpose of favoring a political party, incumbent legislator or member of Congress, or other person or group, or for the purpose of augmenting or diluting the voting strength of a language or racial minority group. In establishing districts, *no use shall be made* of any of the following data:

   a. Addresses of incumbent legislators or members of Congress.
   b. Political affiliations of registered voters.
   c. Previous election results.
   d. Demographic information, other than population head counts, except as required by the Constitution and the laws of the United States.

The section goes on to provide two specific measures of compactness that should be used to compare districts, one of which is the total perimeter of all districts. If the total perimeter is small, then the districts relatively compact.

Contiguity of districts and no reliance on partisan or demographic data are built-in to `redist`. We'll look at how to specify the desired population deviation (no more than 1% by law) in the next section, and discuss compactness in the simulation section.

Setting up the problem in `redist`

In `redist`, a basic redistricting problem is defined by an object of type `redist_map`, which can be constructed using the eponymous function. The user must provide a vector of population counts (defaults to the `pop` column, if one exists) and the desired population parity, and the number of districts. The latter can be inferred if a reference redistricting plan exists. For Iowa, we'll use the adopted 2010 plan as a reference.

```
iowa_map = redist_map(iowa, existing_plan=cd_2010, pop_tol=0.01, total_pop = pop)
print(iowa_map)
#> A <redist_map> with 99 units and 17 fields
#> To be partitioned into 4 districts with population between 761,588.8 - 1.0% and 761,58
8.8 + 1.0%
#> With geometry:
#>     bbox:           xmin: 4081849 ymin: 2879102 xmax: 5834228 ymax: 4024957
#>     projected CRS:  NAD83(HARN) / Iowa North (ftUS)
#> # A tibble: 99 × 17
#>    fips  name  cd_2010    pop  white black  hisp    vap  wvap  bvap  hvap tot_08
#>  * <chr> <chr>   <int>  <dbl>  <dbl> <dbl> <dbl>  <dbl> <dbl> <dbl> <dbl>  <dbl>
#>  1 19001 Adair       3   7682   7507    11   101   5957  5860     5    53   4053
#>  2 19003 Adams       3   4029   3922     8    37   3180  3109     6    22   2206
#>  3 19005 Alla…       1  14330  13325   109   757  11020 10430    82   425   7059
#>  4 19007 Appa…       2  12887  12470    55   181   9993  9745    40    99   6176
#>  5 19009 Audu…       4   6119   6007     9    37   4780  4714     5    27   3435
#>  6 19011 Bent…       1  26076  25387    93   275  19430 19068    49   155  13712
#>  7 19013 Blac…       1 131090 109968 11493  4907 102594 89541  7677  2865  64775
#>  8 19015 Boone       4  26306  25194   202   505  20027 19448   103   260  13929
#>  9 19017 Brem…       1  24276  23459   186   239  18763 18242   155   137  12871
#> 10 19019 Buch…       1  20958  20344    59   243  15282 14979    32   128  10338
#> # ℹ 89 more rows
#> # ℹ 5 more variables: dem_08 <dbl>, rep_08 <dbl>, region <chr>,
#> #   geometry <MULTIPOLYGON [US_survey_foot]>, adj <list>
```

This looks much the same as `iowa` itself, but metadata has been added, and there's a new column, `adj`.

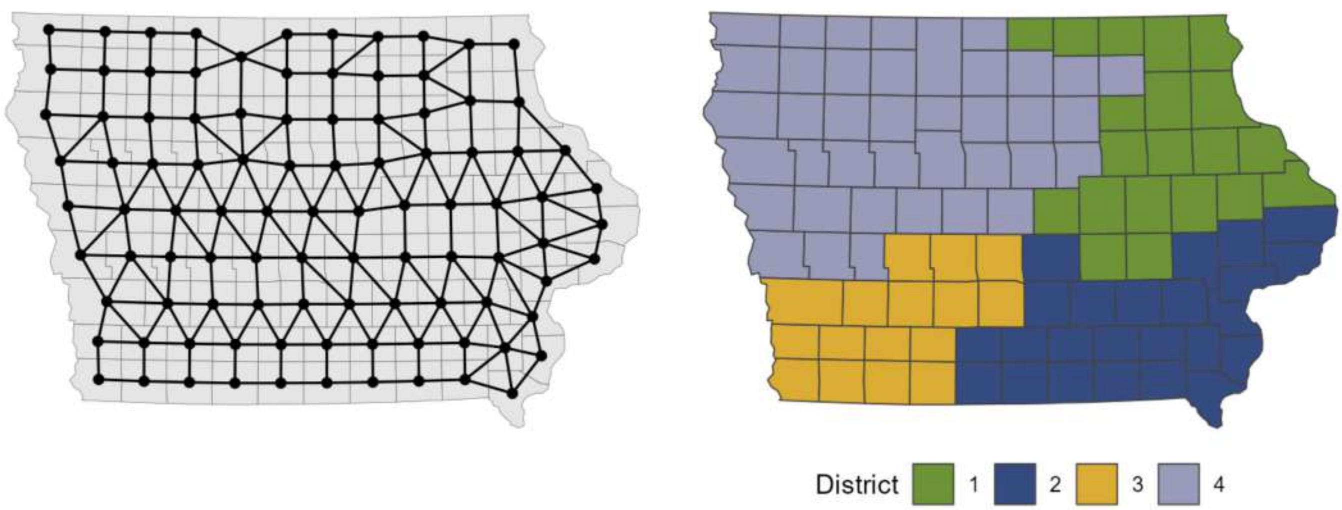
Adjacency-based redistricting

All redistricting algorithms operate on an *adjacency graph*, which is constructed from the actual precinct or county geography. In the adjacency graph, every precinct or county is a node, and two nodes are connected by an edge if the corresponding precincts are geographically adjacent.[7] Creating a contiguous set of districts as part of a redistricting plan then corresponds to creating a *partition* (https://en.wikipedia.org/wiki/Graph_partition) of the adjacency graph.

The `redist_map()` function automatically computes the adjacency graph from the provided shapefile (though one can be provided directly as well), and stores it in the `adj` column as an *adjacency list*, which is, for each precinct, a list of neighboring precincts. As part of this process, the adjacency graph is checked for global contiguity (no islands), which is necessary for the redistricting algorithms to function properly.

We can visualize the adjacency graph by plotting the `redist_map` object.

```
plot(iowa_map, adj=T) + plot(iowa_map)
```



Pre-processing

Often, we want to only analyze a portion of a map, or hold some districts fixed while others are re-simulated. We may also want to implement a status-quo-type constraint that encourages simulated districts to be close to a reference plan. This can be accomplished by freezing the "cores" of each district.

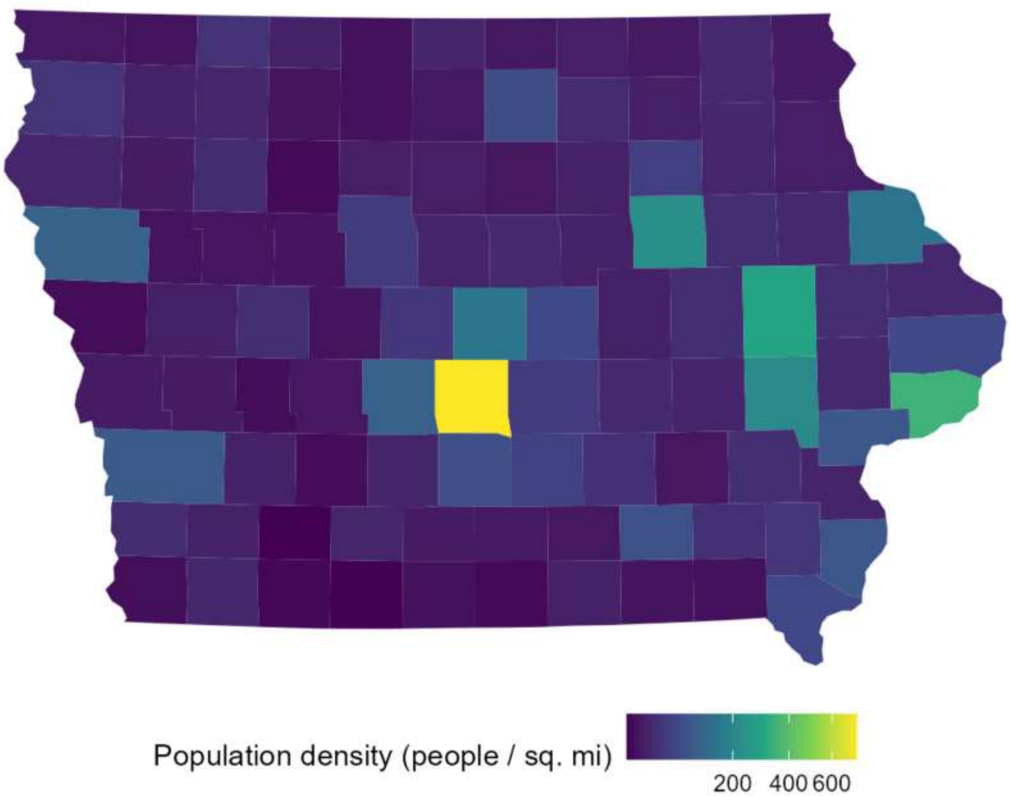
All of these operations fall under the umbrella of map pre-processing, and `redist` is well-equipped to handle them. You can use familiar `dplyr` verbs like `filter()` and `summarize()`, along with new `redist` operations like `freeze()`, `make_cores()`, and `merge_by()`, to operate on `redist_map` objects. The package will make the appropriate modifications to the geometry and adjacency graph in the background.

The map pre-processing vignette (map-preproc.html) contains more information and examples about these operations.

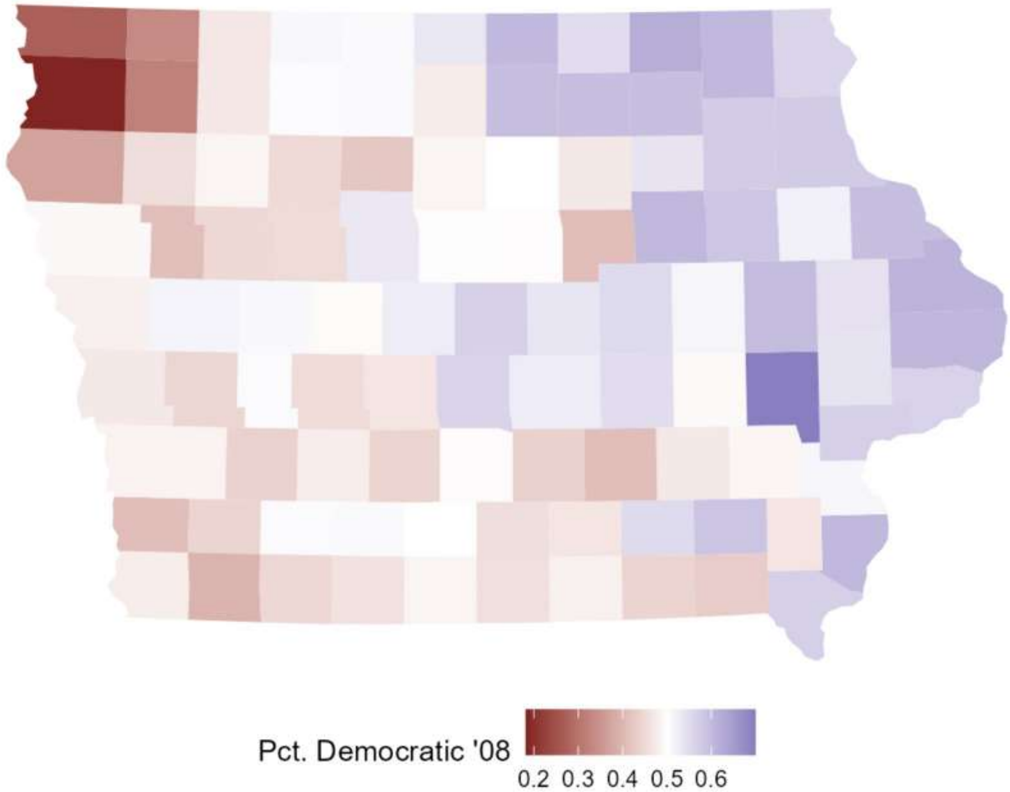
Exploring the geography

To get a feel for the demographic and political geography of the state, we'll make some plots from the `iowa_map` object. We see that the state is mostly rural and white, with Polk county (Des Moines) the largest and densest. Politically, most counties are relatively balanced between Democrats and Republicans (at least in the '08 election), though there is a rough east-west gradient.

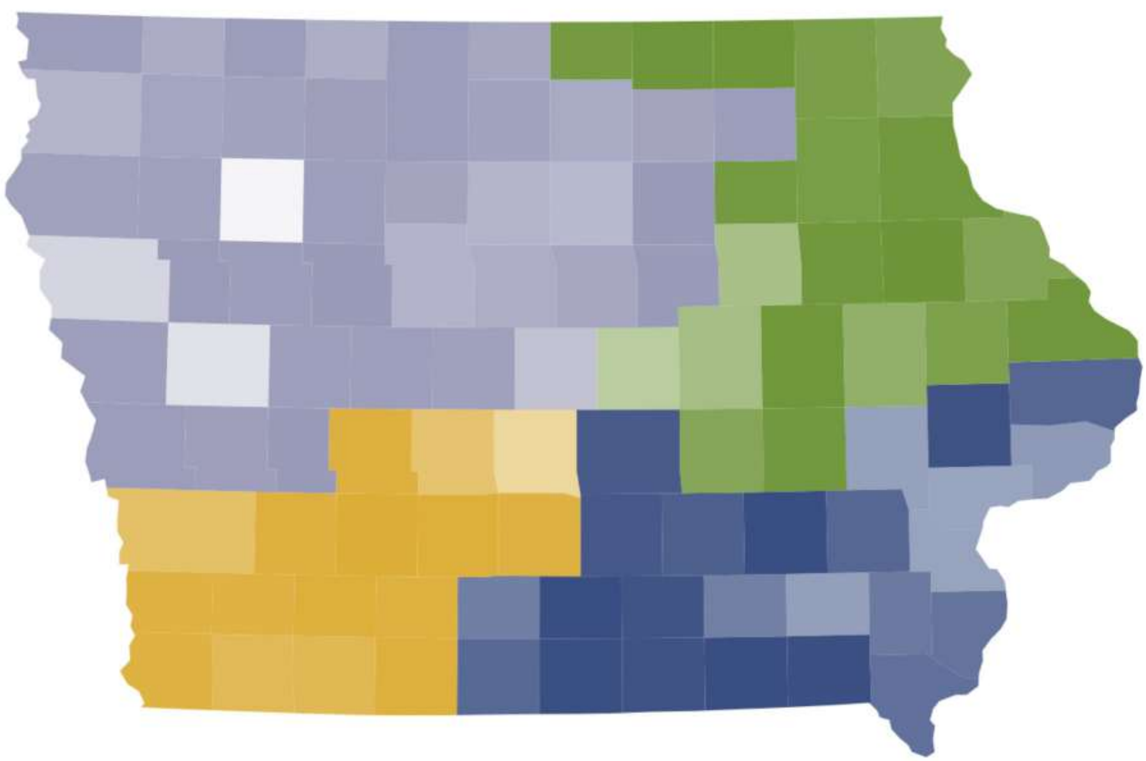
```
areas = as.numeric(units::set_units(sf::st_area(iowa_map$geometry), mi^2))
plot(iowa_map, fill = pop / areas) +
    scale_fill_viridis_c(name="Population density (people / sq. mi)",
                         trans="sqrt")
```




```
plot(iowa_map, fill = dem_08 / tot_08) +
    scale_fill_gradient2(name="Pct. Democratic '08",  midpoint=0.5)
```



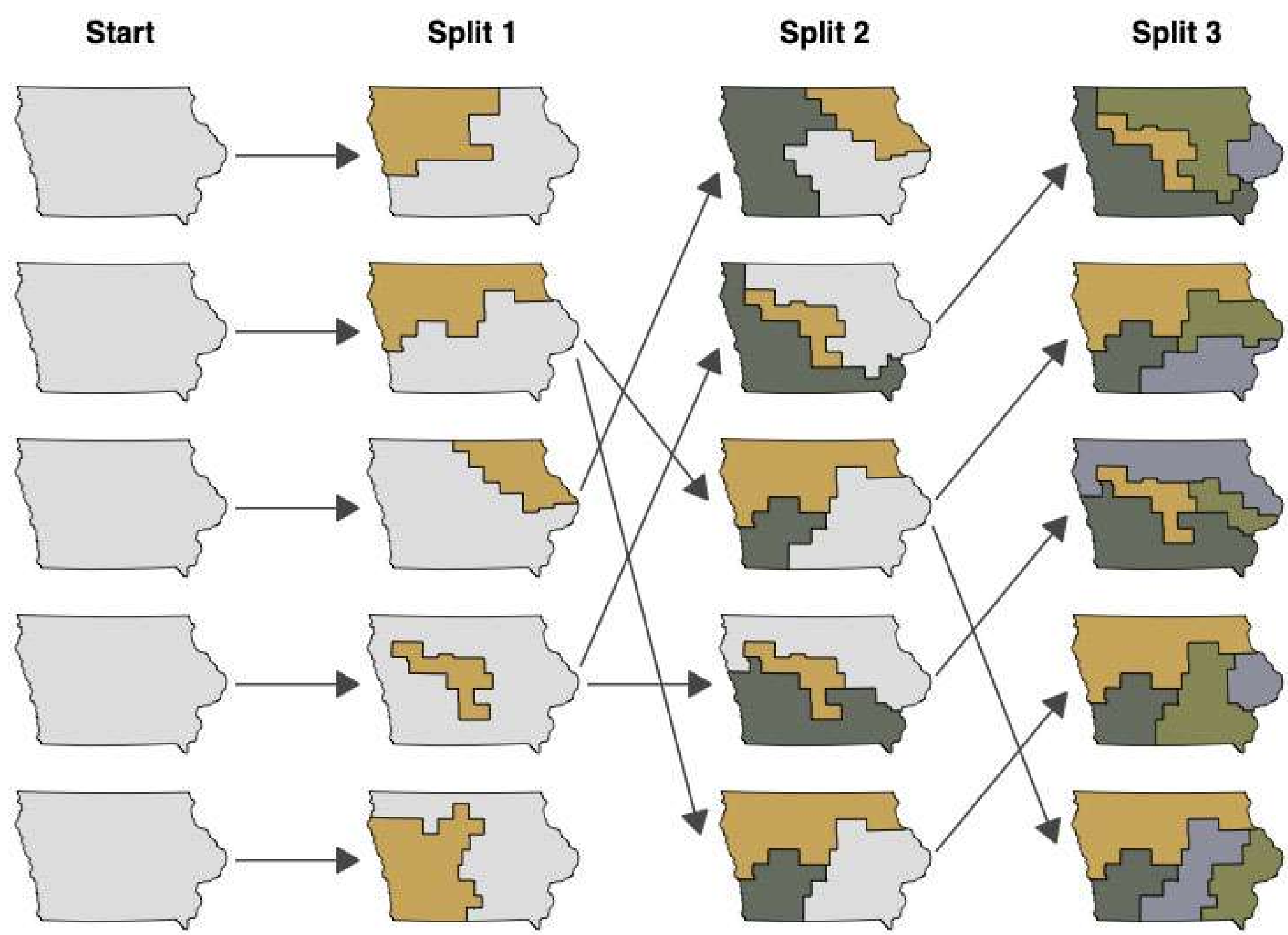

```
plot(iowa_map, fill = wvap / vap, by_distr = TRUE)
```



Simulating redistricting plans

The crux of a redistricting analysis is actually simulating new redistricting plans. As discussed above, `redist` provides several algorithms for performing this simulation, and each has its own advantages and incorporates a different set of constraints. Here, we'll demonstrate use of the `redist_smc()` algorithm, a Sequential Monte Carlo (SMC)-based procedure which is the recommended choice for most redistricting analyses.

SMC creates plans directly, by drawing district boundaries one at a time, as illustrated below.



Because of the way districts are drawn in SMC, the generated districts are relatively compact by default. This can be further controlled by the `compactness` parameter (although `compactness=1` is particularly computationally convenient).

To simulate, we call `redist_smc()` on our `redist_map` object. We provide the `runs=2` parameter, which will run the SMC algorithm twice, in parallel. This doubles the total number of sampled plans, but more importantly, it provides extremely valuable diagnostic information about whether the algorithm is sampling properly.

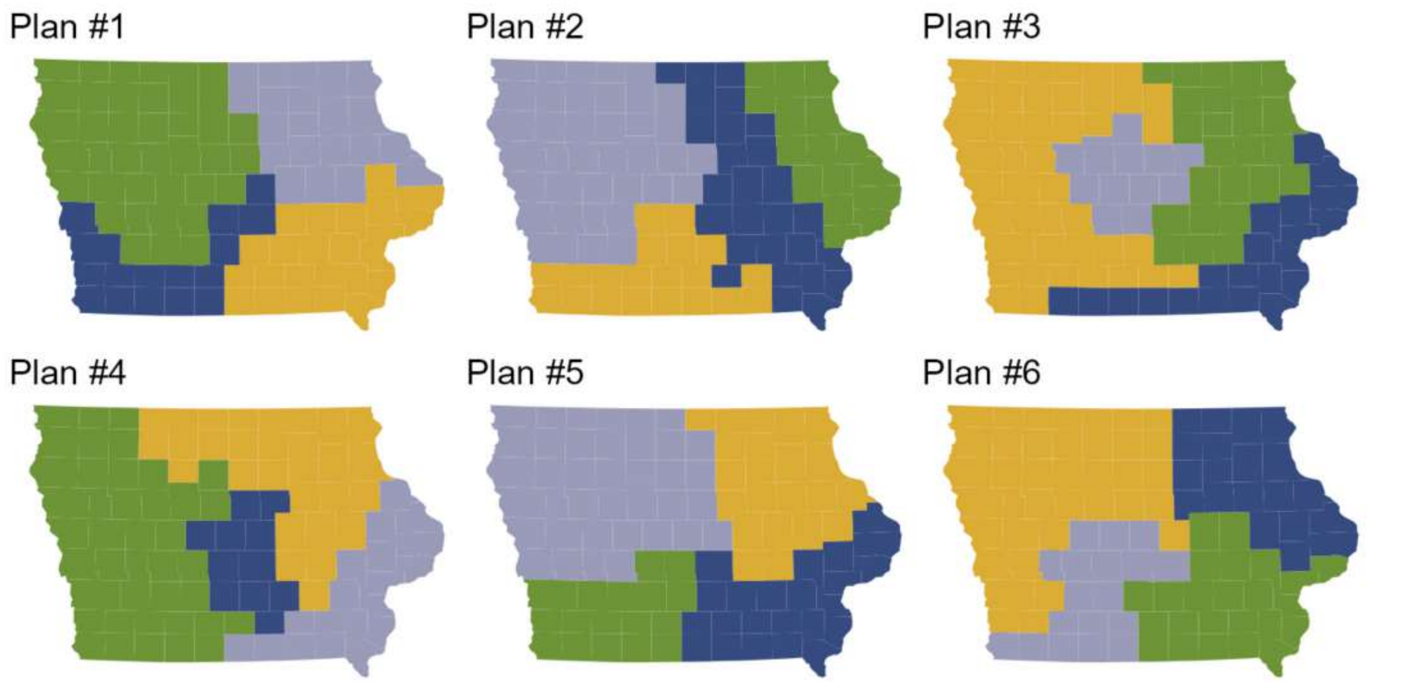
```
iowa_plans = redist_smc(iowa_map, 500, compactness=1, runs=2)
```

The output from the algorithm is a `redist_plans` object, which stores a matrix of district assignments for each precinct and simulated plans, and a table of summary statistics for each district and simulated plan. The existing 2010 plan has also been automatically added as a reference plan. Additional reference or comparison plans may be added using `add_reference()`.

```
print(iowa_plans)
#> A <redist_plans> containing 1,000 sampled plans and 1 reference plan
#> Plans have 4 districts from a 99-unit map, and were drawn using Sequential
#> Monte Carlo.
#> With plans resampled from weights
#> Plans matrix: int [1:99, 1:1001] 1 1 2 3 4 2 2 4 2 2 ...
#> # A tibble: 4,004 × 4
#>    draw    district total_pop chain
#>    <fct>      <int>     <dbl> <int>
#>  1 cd_2010        1    761612    NA
#>  2 cd_2010        2    761548    NA
#>  3 cd_2010        3    761624    NA
#>  4 cd_2010        4    761571    NA
#>  5 1              1    760836     1
#>  6 1              2    757367     1
#>  7 1              3    762946     1
#>  8 1              4    765206     1
#>  9 2              1    756071     1
#> 10 2              2    765242     1
#> # ℹ 3,994 more rows
```

We can explore specific simulated plans with `redist.plot.plans()`.

```
redist.plot.plans(iowa_plans, draws=1:6, shp=iowa_map)
```



Analyzing the simulated plans

A `redist_plans` object, the output of a sampling algorithm, links a matrix of precinct assignments to a table of district statistics, and this linkage makes analyzing the output a breeze.

Sometimes it may be useful to renumber the simulated districts (which have random numbers in general) to match the reference plan as closely as possible. This adds a `pop_overlap` column which measures how much of the population is in the same district in both a given plan and the reference plan.

```
iowa_plans = match_numbers(iowa_plans, iowa_map$cd_2010)
print(iowa_plans)
#> A <redist_plans> containing 1,000 sampled plans and 1 reference plan
#> Plans have 4 districts from a 99-unit map, and were drawn using Sequential
#> Monte Carlo.
#> With plans resampled from weights
#> Plans matrix: int [1:99, 1:1001] 3 3 1 2 4 1 1 4 1 1 ...
#> # A tibble: 4,004 × 5
#>    draw    district total_pop chain pop_overlap
#>    <fct>   <ord>        <dbl> <int>       <dbl>
#>  1 cd_2010 1           761548    NA       1
#>  2 cd_2010 2           761624    NA       1
#>  3 cd_2010 3           761612    NA       1
#>  4 cd_2010 4           761571    NA       1
#>  5 1       1           765206     1       0.875
#>  6 1       2           762946     1       0.875
#>  7 1       3           757367     1       0.875
#>  8 1       4           760836     1       0.875
#>  9 2       1           756071     1       0.691
#> 10 2       2           765242     1       0.691
#> # ℹ 3,994 more rows
```

Then we can add summary statistics by district, using `redist`'s analysis functions. Here, we'll compute the population deviation, the perimeter-based compactness measure related to the Iowa Code's redistricting requirements, and the fraction of minority voters and two-party Democratic vote share by district.

```
county_perims = prep_perims(iowa_map, iowa_map$adj)

iowa_plans = iowa_plans %>%
    mutate(pop_dev = abs(total_pop / get_target(iowa_map) - 1),
           comp = comp_polsby(pl(), iowa_map, perim_df=county_perims),
           pct_min = group_frac(iowa_map, vap - wvap, vap),
           pct_dem = group_frac(iowa_map, dem_08, dem_08 + rep_08))
print(iowa_plans)
#> With plans resampled from weights
#> Plans matrix: int [1:99, 1:1001] 3 3 1 2 4 1 1 4 1 1 ...
#> # A tibble: 4,004 × 9
#>    draw    district total_pop chain pop_overlap   pop_dev  comp pct_min pct_dem
#>    <fct>   <ord>        <dbl> <int>       <dbl>     <dbl> <dbl>   <dbl>   <dbl>
#>  1 cd_2010 1           761548    NA       1     0.0000535 0.302  0.0737   0.592
#>  2 cd_2010 2           761624    NA       1     0.0000463 0.360  0.0968   0.579
#>  3 cd_2010 3           761612    NA       1     0.0000305 0.529  0.114    0.531
#>  4 cd_2010 4           761571    NA       1     0.0000233 0.522  0.0788   0.491
#>  5 1       1           765206     1       0.875 0.00475   0.468  0.0665   0.594
#>  6 1       2           762946     1       0.875 0.00178   0.439  0.0963   0.579
#>  7 1       3           757367     1       0.875 0.00554   0.247  0.118    0.541
#>  8 1       4           760836     1       0.875 0.000988  0.588  0.0831   0.477
#>  9 2       1           756071     1       0.691 0.00725   0.481  0.0791   0.594
#> 10 2       2           765242     1       0.691 0.00480   0.274  0.0895   0.585
#> # ℹ 3,994 more rows
```

Once summary statistics of interest have been calculated, it’s very important to check the algorithm’s diagnostics. As with any complex sampling algorithm, things can go wrong. Diagnostics, while not flawless, can help catch problems and stop you from making conclusions that are actually the fault of a broken sampling process. The `summary()` function is `redist` ’s one-stop-shop for algorithm diagnostics.

```
summary(iowa_plans)
#> SMC: 1,000 sampled plans of 4 districts on 99 units
#> `adapt_k_thresh`=0.99 • `seq_alpha`=0.5
#> `est_label_mult`=1 • `pop_temper`=0
#> Plan diversity 80% range: 0.45 to 0.81
#>
#> R-hat values for summary statistics:
#> pop_overlap     pop_dev        comp     pct_min     pct_dem
#>       1.002       1.014       1.033       1.001       1.014
#> Sampling diagnostics for SMC run 1 of 2 (500 samples)
#>          Eff. samples (%) Acc. rate Log wgt. sd  Max. unique Est. k
#> Split 1       492 (98.4%)      5.6%        0.25   316 (100%)      5
#> Split 2       484 (96.8%)      7.5%        0.36   304 ( 96%)      4
#> Split 3       476 (95.1%)      3.0%        0.44   272 ( 86%)      3
#> Resample      402 (80.5%)       NA%        0.43   407 (129%)     NA
#> Sampling diagnostics for SMC run 2 of 2 (500 samples)
#>          Eff. samples (%) Acc. rate Log wgt. sd  Max. unique Est. k
#> Split 1       491 (98.3%)      5.6%        0.26   309 ( 98%)      5
#> Split 2       484 (96.8%)      6.8%        0.36   297 ( 94%)      4
#> Split 3       480 (96.1%)      2.9%        0.42   264 ( 84%)      3
#> Resample      425 (85.0%)       NA%        0.39   424 (134%)     NA
#> •  Watch out for low effective samples, very low acceptance rates (less than
#> 1%), large std. devs. of the log weights (more than 3 or so), and low numbers
#> of unique plans. R-hat values for summary statistics should be between 1 and
#> 1.05.
```

There's a lot of diagnostic output there, which you should read more about with `?summary.redist_plans`. If anything is obviously wrong, the function will alert you and provide instructions on how to try to fix it. But these warnings aren't flawless, and it's important to check the numbers yourself.

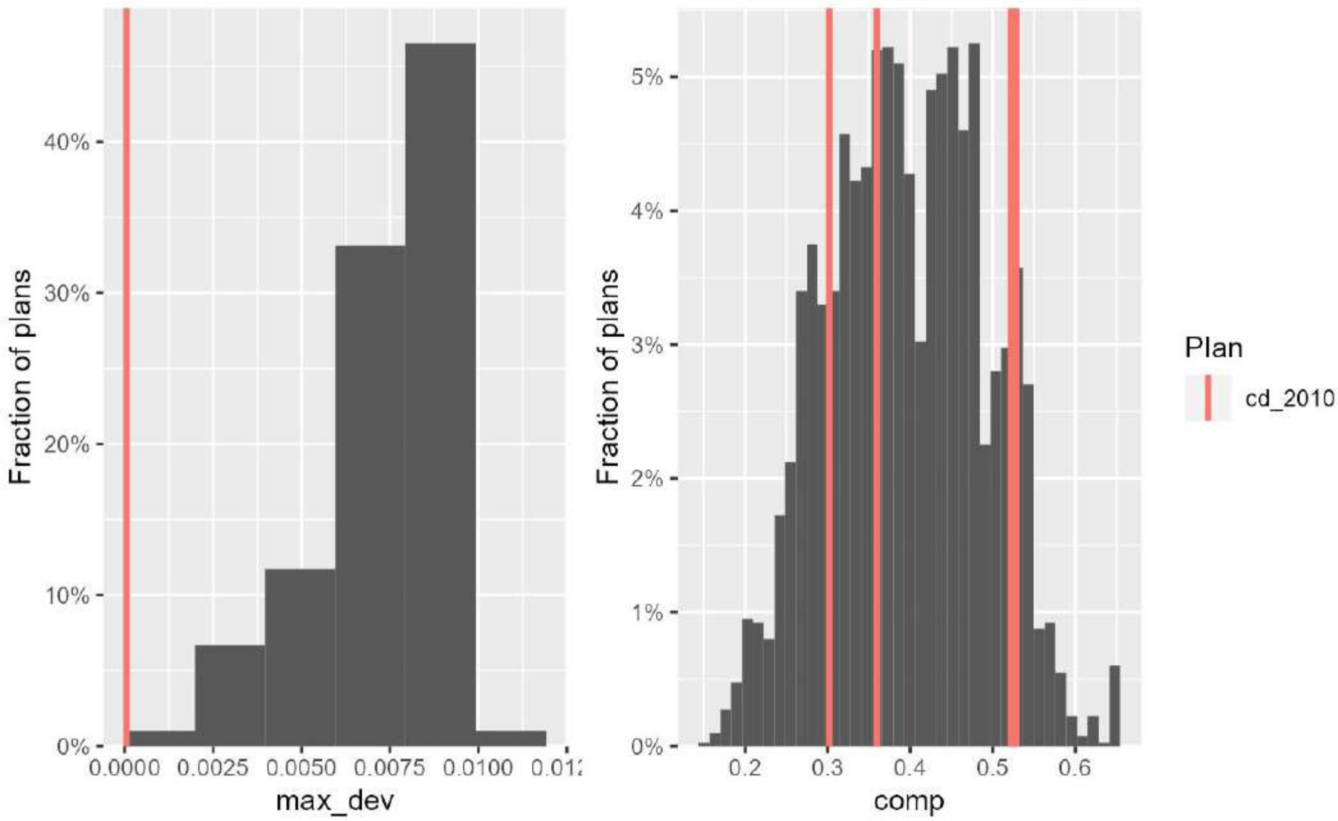
If you've used 2 or more `runs`, as we have, `summary()` will calculate R-hat values. These should be as close to 1 as possible, and generally less than 1.05. If they are bigger than that, it means that multiple independent runs of the algorithm gave different results. More samples (higher `nsims`) are usually called for. The other number to keep an eye on is the plan diversity (top of the output), whose 80% range should generally cover the range from 0.5–0.8.

Since our diagnostics look good, we can return to our analysis. It's quick to turn district-level statistics from a `redist_plans` object into plan-level summary statistics.

```
plan_sum = group_by(iowa_plans, draw) %>%
    summarize(max_dev = max(pop_dev),
              avg_comp = mean(comp),
              max_pct_min = max(pct_min),
              dem_distr = sum(pct_dem > 0.5))
print(plan_sum)
#> A <redist_plans> containing 1,000 sampled plans and 1 reference plan
#> Plans have 4 districts from a 99-unit map, and were drawn using Sequential
#> Monte Carlo.
#> With plans resampled from weights
#> Plans matrix: int [1:99, 1:1001] 3 3 1 2 4 1 1 4 1 1 ...
#> # A tibble: 1,001 × 5
#>    draw      max_dev avg_comp max_pct_min dem_distr
#>    <fct>       <dbl>    <dbl>       <dbl>     <int>
#>  1 cd_2010 0.0000535    0.428       0.114         3
#>  2 1       0.00554      0.435       0.118         3
#>  3 2       0.00725      0.413       0.113         3
#>  4 3       0.00997      0.330       0.128         3
#>  5 4       0.00457      0.365       0.121         3
#>  6 5       0.00894      0.508       0.115         3
#>  7 6       0.00889      0.406       0.119         3
#>  8 7       0.00562      0.350       0.117         3
#>  9 8       0.00921      0.407       0.110         3
#> 10 9       0.00984      0.375       0.119         3
#> # ℹ 991 more rows
```

These tables of statistics are easily plotted using existing libraries like `ggplot2`, but `redist` provides a number of helpful plotting functions that automate some common tasks, like adding a reference line for the reference plan. The output of these functions is a `ggplot` object, allowing for further customization.

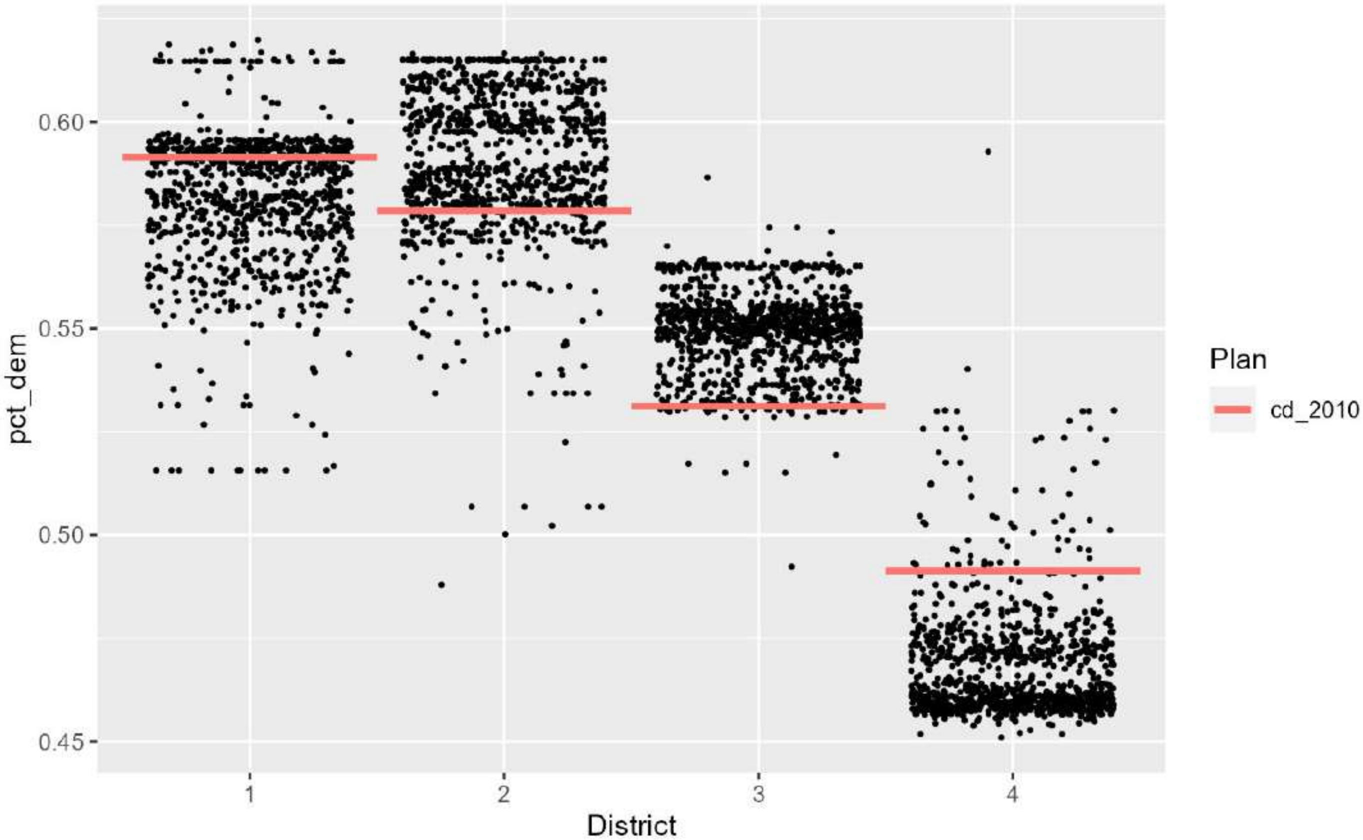
```
library(patchwork)

hist(plan_sum, max_dev) + hist(iowa_plans, comp) +
    plot_layout(guides="collect")
```



We can see that the adopted plan has nearly complete population parity, and that its districts are roughly as compact on average as those simulated by the SMC algorithm.

One of the most common, and useful, plots, for studying the partisan characteristics of a plan, is to plot the fraction of a group (or party) within each district, and compare to the reference plan. Generally, we would first sort the districts by this quantity first, to make the numbers line up, but here we've already renumbered the districts to match the reference plan as closely as possible.

```
plot(iowa_plans, pct_dem, sort=FALSE, size=0.5)
```

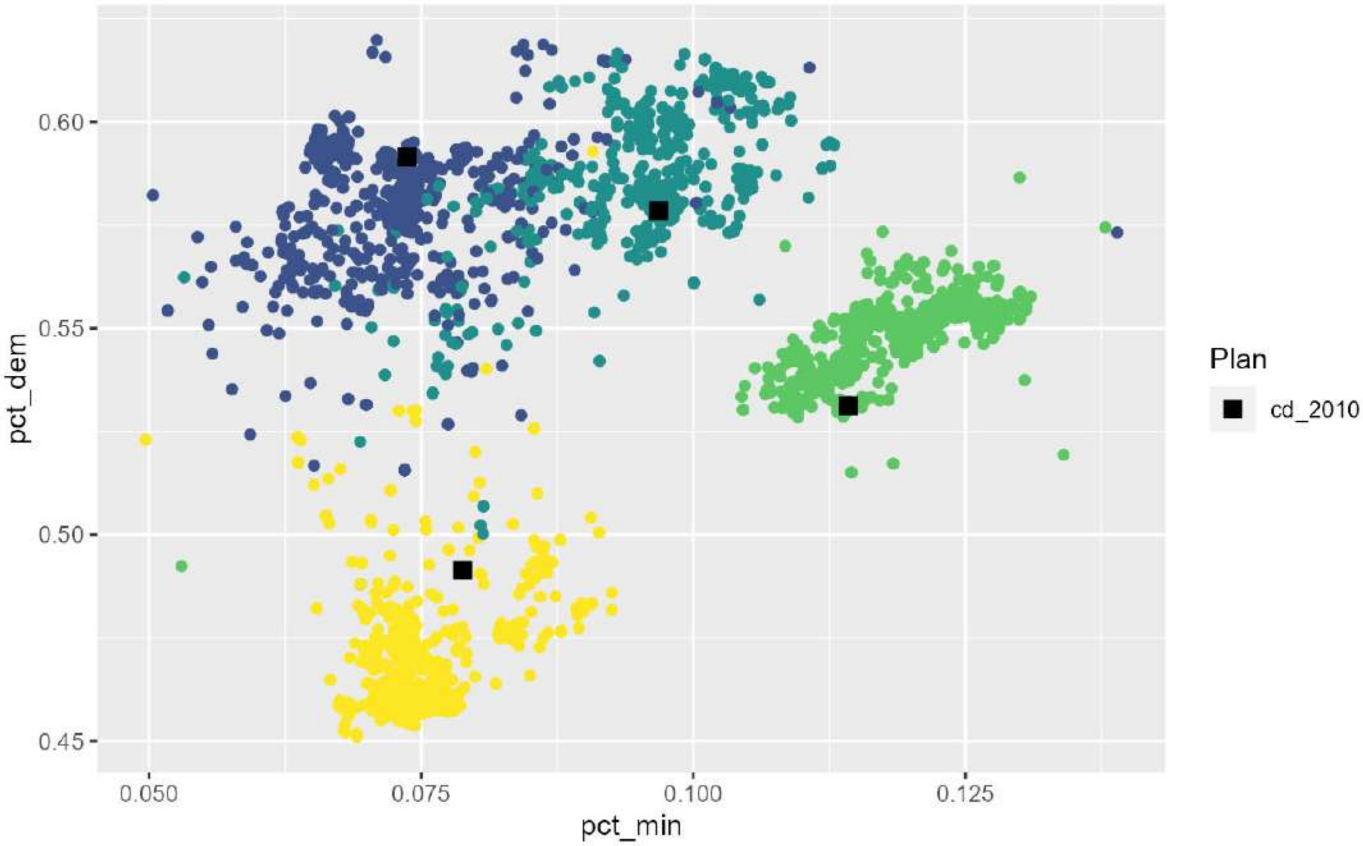


We see that districts 1 and 2 look normal, but it appears that, relative to our ensemble, district 4 (NW Iowa) is more Democratic, and district 3 (SW Iowa, Des Moines) is less Democratic. However, the reference plan does not appear to be a complete outlier.

We might also want to look at how the Democratic fraction in each district compares to the fraction of minority voters. We can make a scatterplot of districts, and overlay the reference districts, using `redist.plot.scatter`. We'll also color by the district number (higher numbers are in lighter colors).

Once again, we see that while district 1 and 2 of the reference plan look normal, district has a lower number of Democrats and minority voters than would otherwise be expected.

```
pal = scales::viridis_pal()(5)[-1]
redist.plot.scatter(iowa_plans, pct_min, pct_dem,
                    color=pal[subset_sampled(iowa_plans)$district]) +
    scale_color_manual(values="black")
```



From here, it is easy to keep exploring, using the functionality of `redist_plans` and the built-in plotting functions. More complex model-based analyses could also be performed using the district-level or plan-level statistics.

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1. from Sequential Monte Carlo for Sampling Balanced and Compact Redistricting Plans (https://arxiv.org/pdf/2008.06131.pdf)↩
2. based on Carter, D., Herschlag, G., Hunter, Z., and Mattingly, J. (2019). A merge-split proposal for reversible Monte Carlo Markov chain sampling of redistricting plans. arXiv preprint arXiv:1911.01503.↩
3. from Automated Redistricting Simulation Using Markov Chain Monte Carlo (https://doi.org/10.1080/10618600.2020.1739532) *Journal of Computational and Graphical Statistics*↩
4. from The Essential Role of Empirical Validation in Legislative Redistricting Simulation (https://imai.fas.harvard.edu/research/files/enumerate.pdf)↩
5. from Cannon, S., Goldbloom-Helzner, A., Gupta, V., Matthews, J. N., & Suwal, B. (2020). Voting Rights, Markov Chains, and Optimization by Short Bursts. arXiv preprint arXiv:2011.02288.↩
6. from Jowei Chen and Jonathan Rodden (2013) “Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures.” Quarterly Journal of Political Science. 8(3): 239-269.↩
7. for `redist`’s purposes, adjacency requires that two regions touch at more than just one point or corner.↩

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