IN THE SUPREME COURT OF WISCONSIN

No. 2021AP001450 OA

BILLIE JOHNSON, ERIC O'KEEFE, ED PERKINS and RONALD ZAHN,

Petitioners,

BLACK LEADERS ORGANIZING FOR COMMUNITIES, VOCES DE LA FRONTERA, LEAGUE OF WOMEN VOTERS OF WISCONSIN, CINDY FALLONA, LAUREN STEPHENSON, REBECCA ALWIN, CONGRESSMAN GLENN GROTHMAN, CONGRESSMAN MIKE GALLAGHER, CONGRESSMAN BRYAN STEIL, CONGRESSMAN TOM TIFFANY, CONGRESSMAN SCOTT FITZGERALD, LISA HUNTER, JACOB ZABEL, JENNIFER OH, JOHN PERSA, GERALDINE SCHERTZ, KATHLEEN QUALHEIM, GARY KRENZ, SARAH J. HAMILTON, STEPHEN JOSEPH WRIGHT, JEAN-LUC THIFFEAULT, and SOMESH JHA,

Intervenors-Petitioners,

v.

WISCONSIN ELECTIONS COMMISSION, MARGE BOSTELMANN in her official capacity as a member of the Wisconsin Elections Commission, JULIE GLANCEY in her official capacity as a member of the Wisconsin Elections Commission, ANN JACOBS in her official capacity as a member of the Wisconsin Elections Commission, DEAN KNUDSON in his official capacity as a member of the Wisconsin Elections Commission, ROBERT SPINDELL, JR. in his official capacity as a member of the Wisconsin Elections Commission and MARK THOMSEN in his official capacity as a member of the Wisconsin Elections Commission,

Respondents,

THE WISCONSIN LEGISLATURE, GOVERNOR TONY EVERS, in his official capacity, and JANET BEWLEY SENATE DEMOCRATIC MINORITY LEADER, on behalf of the Senate Democratic Caucus,

Intervenors-Respondents.

EXPERT REPORT OF DR. DARYL DEFORD ON BEHALF OF INTERVENORS-PETITIONERS CITIZEN MATHEMATICIANS AND SCIENTISTS

Expert Report of Daryl R. DeFord on behalf of the Citizen Mathematicians and Scientists

December 30, 2021

I Qualifications

I am an Assistant Professor of Data Analytics in the Department of Mathematics and Statistics at Washington State University. I earned A.M. and Ph.D. degrees in Mathematics at Dartmouth College and also hold a B.S. in Theoretical Mathematics from Washington State University. From 2018-2020 I was a postdoctoral associate in the Geometric Data Processing Group in the Computer Science and Artificial Intelligence Laboratory at the Massachusetts Institute of Technology and affiliated with the Metric Geometry and Gerrymandering Group in the Jonathan M. Tisch College of Civic Life at Tufts University with a full-time focus on computational redistricting research.

My mathematical work focuses on applications of combinatorial and algebraic techniques to the analysis of social data and particularly includes the study of statistical sampling techniques for political redistricting problems. This work includes both theoretical design and analysis of algorithms as well as empirical projects modeling the interactions between districting criteria. My redistricting work has been published in the *Harvard Data Science Review*, *Political Analysis*, *Statistics and Public Policy*, *Journal of Computational Social Science*, *Mathematical Association of America's Math Horizons*, *Physical Review E*, and *Society of Industrial and Applied Mathematics Journal on Applied Algebra and Geometry.* I have given dozens of presentations on computational redistricting and designed an IAP course at MIT on the topic. As a postdoc, I helped supervise the Voting Rights Data Institute summer program in 2018 and 2019 and in Summer 2021 I supervised a team of research fellows through the University of Washington's Data Science for Social Good program applying computational redistricting to initial stages of the map-making process.

During the current redistricting cycle, Dr. Jeanne Clelland, Dr. Beth Malmskog, Dr. Flavia Sancier-Barbosa, and I provided reports and analysis for the 2021 Colorado Independent Legislative Redistricting Commission. Our work was cited by the commission in their final report supporting their maps and the Colorado State Supreme Court cited our work as evidence that the commission complied with the legislative requirement to optimize for the number of competitive districts. In 2019, I performed computational work and served as a collaborator on an Amicus brief to the United States Supreme Court for *Rucho v. Common Cause*. I have not previously testified as an expert witness or been deposed in any legal proceeding.

A full copy of my CV is included in Appendix A which contains a list of my publications in the last 10 years. For my work on this matter, I am being compensated at a rate of $300 per hour. This compensation does not depend in any way on the results of my analysis, the conclusions that I draw, or the eventual outcome of the litigation.

II Executive Summary

Counsel for the Citizen Mathematicians and Scientists requested that I review and analyze congressional and state legislative maps and accompanying expert materials submitted in this proceeding. This work involved evaluation of several redistricting criteria identified by counsel, including (but not limited to) population deviation, compliance with metrics related to the Voting Rights Act, respect for counties, towns, and wards, district compactness, and contiguity. It also required evaluating how proposed maps perform on metrics of "least change," including whether proposed maps are consistent with an approach of minimizing changes made to conform malapportioned districts to constitutional and statutory requirements. In connection with the work, I assessed how proposed least-change maps that comply with constitutional and statutory requirements perform on traditional neutral redistricting criteria.

To facilitate analysis, I identified appropriate and reliable quantitative metrics, which are laid out in Part IV of this report. Because most of the metrics are not binary, but rather measured on a spectrum ("continuous-valued"), it is possible to evaluate the relative performance of the proposed maps. Additionally, where appropriate, quantitative thresholds can be applied to disqualify maps that do not perform adequately on a continuous-valued metric. For example, although population deviation is continuous-valued, one can apply a quantitative threshold to evaluate whether proposed congressional maps comply with legal requirements regarding population equality.

II.A Congressional Maps

I reviewed Congressional plans submitted by the Congressman, Governor, and Hunter Plaintiffs, in addition to the 2011 enacted plan and the plan proposed by the Citizen Mathematicians and Scientists (the "CMS plan"). In doing so, I applied my understanding of the law applicable to congressional redistricting in Wisconsin, including: (i) the requirement that plans contain equally populated districts except where practically impossible to create them; (ii) the requirement that plans comply with the Voting Rights Act of 1965; (iii) the direction that proposed maps adopt a least-change approach to bringing malapportioned maps into compliance; and (iv) the importance of traditional neutral districting criteria in distinguishing between plans that satisfy legal requirements and implement a least-change approach. My complete analysis of the Congressional plans with respect to these metrics is described in Section V.B and here I summarize my conclusions.

Among the alternatives to the CMS congressional map that I was instructed to analyze, and applying my understanding of the law applicable to Wisconsin congressional redistricting, I conclude that the Legislature's map performs better than other alternatives to the CMS plan on federal requirements, that the Governor's map performs better than other alternatives to the CMS plan on metrics of least change, and that each party performs better than other alternatives to the CMS plan on at least one criterion that is considered traditional and neutral when evaluating congressional maps. However, I further conclude that the CMS plan performs just as well as the Legislature's map on federal requirements, outperforms or equals the Governor on some metrics of least change, and performs best on each of the traditional neutral criteria considered in this report. Accordingly, after reviewing and analyzing alternatives to the CMS plan, I am not able to identify any that performs as effectively under the applicable framework.

II.B State Legislative Maps

I reviewed the Bewley, BLOC, Governor, Hunter, and Legislature state legislative plans, as well as the 2011 enacted plans and the CMS state legislative plans. In doing so, I applied my understanding of the law applicable to state legislative redistricting in Wisconsin, including: (i) the requirement that districts approximate population equality across districts as closely as possible; (ii) the requirement that plans comply with the Voting Rights Act of 1965; (iii) the requirement that plans respect county, town, and ward lines when reapportioning districts; (iv) the requirement that plans create legislative districts "in as compact form as practicable"; (v) the requirement that districts be contiguous; (vi) the direction that plans adopt a least-change approach to bringing malapportioned maps into compliance with constitutional and statutory requirements; and (vii) the importance of traditional neutral districting criteria in distinguishing between plans that satisfy legal requirements and implement a least-change approach. My complete analysis of the state legislative plans is described in Section V.C-D and here I summarize my conclusions.

With respect to Senate districts, and among alternatives to the CMS plan, there is significant variance in performance across metrics related to constitutional and statutory requirements, and on metrics of least change. Having analyzed the performance of each Senate plan across metrics related to constitutional and statutory requirements, I conclude that the plans fall short of the CMS Senate map, which makes necessary modifications to district boundaries to achieve lower population deviation, fewer county splits, and better mean Reock scores than any other Senate proposal, while also performing as well as other plans on metrics related to VRA compliance and the preservation of wards, and performing better than nearly all maps on several important measures of compactness, including mean Polsby-Popper, mean Convex Hull, and cut edges.

With respect to Assembly districts, my conclusions are similar, a logical result given that the Senate districts are composed of nested Assembly districts. In the context of Assembly districts, and among alternatives to the CMS plan (and only among those alternatives), I again find considerable variance in performance across metrics related to constitutional and statutory requirements, and on metrics of least change. Based on my analysis of each Assembly plan across metrics related to those constitutional and statutory requirements, I conclude that the plans again fall short of the CMS map, which makes necessary modifications to district boundaries to achieve the lowest population deviation, the fewest county splits, and the second best score on critical measures of compactness, including mean Reock, mean Polsby-Popper, mean Convex Hull, and cut edges. The CMS plan accomplishes all of that without splitting a single ward, and while performing effectively on metrics related to VRA compliance.

III Assignment

Counsel for a group of Wisconsin voters (“Citizen Mathematicians and Scientists” or “CMS”) asked me to evaluate proposed maps and supporting expert reports submitted to the Wisconsin Supreme Court (the “Court”) on December 15, 2021.

More specifically, counsel asked that I evaluate congressional maps submitted by three parties, including Congressmen Glen Grothman, Mike Gallagher, Bryan Steil, Tom Tiffany, and Scott Fitzgerald (together, the “Congressmen”),[1] Governor Tony Evers (the “Governor”), and individual plaintiffs Lisa Hunter, Jacob Zabel, Jennifer Oh, John Persa, Geraldine Schertz, and Kathleen Qualheim (together, “Hunter” or “Hunter plaintiffs”).

Counsel also asked that I evaluate state legislative maps submitted by five parties: the Wisconsin Legislature (the “Legislature”), the Governor, Senate Minority Leader Janet Bewley (“Bewley”), Black Leaders Organizing for Communities, Voces de la Frontera, League of Women Voters of Wisconsin, Cindy Fallona, Lauren Stephenson, and Rebecca Alwin (together, “BLOC” or “BLOC plaintiffs”), and the Hunter plaintiffs.

I also compared these maps with the congressional and state legislative maps submitted by the Citizen Mathematicians and Scientists. As several of the reports and briefs make comparisons with the 2011 enacted plan, I also provide summary metrics for those maps.

For each of the proposed maps, I was asked to evaluate the following:

- How the proposed maps perform on redistricting criteria that are mandated by law, such as equal population, compliance with the Voting Rights Act, and—in the case of state legislative plans—respect for counties, towns, and wards, district compactness, and district contiguity.

[1]The Congressmen propose that the Court adopt a congressional map passed by the Legislature.

- How the proposed maps perform on various metrics of "least change" and effectuate the Court's direction that parties minimize changes made to conform the currently malapportioned maps to constitutional and statutory requirements.
- How proposed least-change maps that comply with constitutional and statutory requirements perform on various traditional neutral redistricting criteria.

To carry out this task, I measured the properties of each redistricting proposal, and the existing malapportioned maps, according to several different metrics. My conclusions are summarized in Part II of this report. The measures, metrics, methodologies, and data that I use are defined in Part IV of this report. My more granular analysis of the parties' congressional plans is set out in Part V.B of this report. And my analysis of the parties' state legislative plans is set out in Part V.C-D of this report.

In addition to measuring the properties of each redistricting proposal, I also evaluate the claims made in expert reports or affidavits submitted with the proposals. These include the expert reports or affidavits submitted by Dr. Loren Collingwood, Dr. David T. Canon, Dr. Kenneth R. Mayer, Tom Schreibel, Dr. Jeanne Clelland, Dr. Steph用 Ansolabehere, Dr. Brian Amos, Dr. James G. Gimpel, Dr John Alford, and Thomas M. Bryan.

In measuring the properties of each redistricting proposal and evaluating claims made in expert reports or affidavits accompanying them, I relied on the briefs, maps, and reports submitted to the Court on December 15, 2021. I also relied on additional materials that the litigants produced to each other after making their December 15, 2021 submissions. Finally, I relied on data obtained from the Wisconsin Legislative Technology Services Bureau (LTSB) and the U.S. Census Bureau. Further description of this data, and a complete list of all materials that I considered in connection with the preparation of this Report, is provided in Appendix B, attached to this report.

IV Redistricting Criteria and Metrics

In order to evaluate the proposed Congressional and state legislative maps submitted in this proceeding, I analyzed performance on several redistricting criteria identified by counsel, including (but not limited to) population deviation, compliance with metrics related to the Voting Rights Act, respect for counties, towns, and wards, district compactness, and contiguity. In this section, I set out the quantitative metrics, used to conduct my analysis.

In reviewing the metrics, it is important to remember that the relationship between them can be complex, and attempting to improve performance on one of those criterion often involves diminishing performance on another. Throughout this report, I highlight instances where criteria are in tension and how that affects my analysis of the proposed plans.

IV.A Population Deviation

The Court's November 30, 2021 order ("Order") reflects the importance of population deviation. See also Bryan Report ¶ 25. It instructs that congressional districts should be zero-balanced, Order ¶ 25, and that for legislative district populations "there should be as close an approximation to exactness as possible." Order ¶28. To assess compliance with these population deviation requirements, I employ an optimization perspective where plans with smaller top-to-bottom deviations are regarded more favorably. This value, obtained by subtracting the smallest district population from the largest is known as the 'maximum deviation' and is a common measure of the overall population balance of a map.

The guiding legislative and judicial text governing population deviation is usually formulated as an optimization constraint where plans with smaller top-to-bottom deviations are regarded more favorably. This value, obtained by subtracting the smallest district population from the largest is known as the 'maximum deviation' and is a common measure of the overall population balance of a map. For Congressional districts the requirement to minimize this value is frequently referred to as 'zero-balancing', in that there should be a maximum of one person deviation between the largest and smallest district populations. This is different than deviation in either direction from the ideal value by one person, which is a distinction which matters for two of the proposed maps discussed in Section V.B.1 below.

For state legislative districts there is not a tradition of strict zero–balancing between the districts but previous maps implemented in Wisconsin have had deviations of less than 2%. Several of the reports appeal to nationally applied 10% or 2% standards for Assembly and Senate districts but as the evaluated proposals demonstrate, it is possible to do significantly better than these bounds, without compromising other legal or traditional principles.

IV.B Voting Rights Act of 1965

The Voting Rights Act of 1965 (VRA) imposes an additional federal requirement on redistricting plans, requiring in Section 2 that lines cannot be drawn to deny racial, ethnic, or language minority voters the opportunity to "to participate in the political process and to elect representatives of their choice." Discussion around this constraint frequently focuses on the construction and existence of 'opportunity districts' that allow groups the ability to elect candidates of their choice. Compliance with the Supreme Court precedent also requires that lines not be drawn with race as the predominant factor but analysis of opportunity districts does not require evidence of intent, only of impact. This means that districts that were previously well-tuned to satisfy VRA concerns could fail to perform effectively in the next redistricting cycle due to shifts in population or voting preferences between groups.

Litigation over VRA violations centers on three Gingles factors, originally derived from the Supreme Court's 1986 ruling in Thornburg v. Gingles and further extended through a significant body of case law. These factors require a demonstration that:

1. it is possible to create a compact majority-minority district of voters belonging to a racial, ethnic, or language minority within the state,
2. members of the identified community tend to vote as a block for the same type of candidates,
3. and that the remainder of the community tends to vote as a block for a different type of candidates.

The combined existence of the conditions described in the final two criteria is known as racially polarized voting and requires the application of statistical inference to determine likely voting behaviors by group. The collection of techniques that are commonly used for this analysis are known as ecological regression and inference, which attempt to estimate the voting propensities for minority groups by analyzing precinct or ward level returns, together with demographic information about the units.

While the Gingles factors are used in court to analyze whether Section 2 of the VRA has been violated by an enacted map, VRA analysis at the line drawing stage does not necessarily require the construction of districts that meet certain thresholds of minority population as in Gingles 1. Instead, there is a focus on whether specific districts are likely to offer an effective opportunity for minority groups to elect their preferred candidates, using historical election data and the same types of statistical methods for evaluating vote polarization between groups. This is becoming increasingly relevant when considering 'coalition districts' or districts drawn that would not have a majority of a single minority group but that might allow multiple groups to band together to elect similarly preferred candidates.

In Wisconsin over the previous decade there is significant evidence of polarized voting between Black and non-Black voters, as computed as a proportion of the voting age population, as well as evidence to a lesser degree of polarization with respect to Hispanic voters. As both of these groups vote predominantly for Democratic candidates in general statewide elections, it is helpful to look at Democratic primary elections to determine voting block behavior and the likelihood of success of preferred candidates for these groups. Specifically, the 2018 Democratic primaries for Governor and Lieutenant Governor, and votes for the candidates Mahlon Mitchell who lost and Mandela Barnes who won.

Based on this analysis, and similar analysis described by Professor Moon Duchin, *See* Duchin Report at 8, I consider a district as effective for Black voters if it satisfies the following conditions:

- A significant proportion of the citizen voting age population is Black,
- with respect to historical voting data the district would have favored the Black candidate of choice in general elections, and
- Mahlon Mitchell winning the district and Mandela Barnes receiving a strong (80%+) majority with respect to the 2018 Democratic primary election data.

Note that with respect to the first criteria there is not a requirement that districts be drawn with a majority of minority voters in order for a district to be considered effective for that group. The Legislature's expert makes this point clearly in paragraphs 24-26 and footnote 9 discussing Cooper v. Harris. Approaches that only seek to maximize percentages of demographic groups beyond specific thresholds are not necessarily directly responsive to the VRA but in the maps analyzed here, each claimed district also passes the effectiveness test described above.

IV.C Boundary Preservation

Although it is "secondary" to population deviation, Order ¶ 34, I am informed that the Wisconsin Constitution requires that parties engaged in state legislative redistricting maximize protection for county, town, and ward lines. I am also informed that Wisconsin courts look at whether parties have minimized the degree to which district lines "split" county, town, and ward lines when evaluating congressional maps. To evaluate how the proposed plans perform on this criteria, I analyze the frequency with which the proposed maps traverse county, municipality, and ward lines.[2]

I begin with counties because these primary units of Wisconsin political geography. See Citizen Mathematicians and Scientists' Dec. 15 Brief at 19-22. In evaluating how proposed maps perform on this criterion, it is important to remember that parties cannot construct population balanced districts that preserve all county boundaries. Moreover, there are several measures that provide valuable information about party performance on this criterion. First, it is possible to measure the number of counties that are split by district lines, demonstrating at a fundamental level the number of counties that are divided. Second, it is possible to measure the number of times that counties are divided by district lines, which ensures that parties are evaluated not only on the number of counties that they split, but also on the number of county pieces that they create.

Finally, particularly for state legislative districts, some counties must be split multiple times because the population of the county is larger than the ideal population size of a district. Counting these splits against proposals can overestimate the count of splits that were due to the discretion of the line drawer, or created as a tradeoff required to satisfy other priorities, rather than those forced to exist in all population balanced maps. Thus, I will also report results for the number of splits and pieces above those necessary to comply with population equality, a measure I call Pop Split [Unit]. To obtain this value I compute the ratio of the county population to the ideal district population and then round up to the next integer.

[2] Towns are one component of municipalities, which also include cities and villages. Although the Wisconsin constitution does not appear to address cities and villages, I report them because they are indistinguishably grouped together in the LTSB data product.

This evaluation of the number of times a county must be split to adhere to population balance requirements also demonstrates a potential source of rigidity in balancing the redistricting constraints. Consider an example of a county like Outagamie whose population was equal to 2.97 Assembly districts in 2010 but has grown to represent 3.2 Assembly districts in 2020. While it would have been possible to have three districts completely contained in Outagamie in the 2011 map, it must be split into at least four parts in a population balanced 2020 map, regardless of least change considerations. A similar scenario is true in La Crosse county, which goes from needing to touch two districts, which is realized in the enacted plan, to having a population slightly larger than two Assembly districts, particularly given the small amount of population deviation that exists in the proposed plans.

Next, I report results for splits of municipalities. Here, I again report (i) the total number of splits, (ii) the total number of pieces, (iii) the total number of splits not necessitated by the requirement of population equality, and (iv) the total number of pieces greater than those necessitated by the requirement of population equality. Finally, I report on preservation of ward boundaries. Here, I confine my analysis to splits, pieces, and the number of wards that are subdivided multiple times. This approach is appropriate because wards as significantly smaller units than counties and municipalities and are all significantly smaller than congressional, senate, and assembly districts.

IV.D Compactness

I am instructed that the Wisconsin Constitution requires that plans create legislative districts "in as compact form as practicable," Wisc. Const. art. IV, § 4, and that Wisconsin courts have historically treated compactness as a traditional redistricting principle when considering Congressional boundaries. Compactness is a measure of geographic or geometric regularity of a district or districting plan, although it is rarely specifically specified in legislation governing redistricting. In this report, I apply several measures of compactness, computed using Python libraries in the epsg:32616 projection [geocompactness, gerrychain]. Three of them—Polsby-Popper,[3] Reock, and cut edges—are elaborated in the report submitted by Professor Moon Duchin. See Duchin Report at 9-10. The fourth, called the convex hull ratio, measures what proportion of the area of the smallest convex shape containing the district is filled by the district.

In evaluating how the proposed plans perform on compactness, I attempt to account for constraints on the ability to improve compactness. For example, measures that depend on the perimeter are determined by the properties of the discrete units, so a map built out of larger structures, like wards instead of blocks, has less flexibility to tune the districts to be compact under these measures. Moreover, since these measures are defined for individual districts, they have to be combined or averaged in some fashion to obtain a score for the whole plan. Consistent with the expert reports filed in this case, I will report mean values for these metrics across all districts in each plan.

There are also measures that apply to the plan as a whole. All continuous measures suffer from some potential distortions due to map projections and other geographic issues [Bar-Natan–Najt–Schutzmann 2020, Solomon and Barnes 2021], so recently some mathematicians have proposed using discrete measures to support compactness claims [Duchin and Tenner 2018]. A common choice for this is the number of cut edges, which represents the count of the number of adjacent units like wards or blocks that are not placed in the same district, which can be viewed as a discrete version of perimeter.

IV.E Contiguity

Contiguity is the principle that states that districts should be connected, usually in the sense that they could be traversed from point to point without needing to leave the district. To evaluate the proposed plans in this proceeding, I apply the definition of contiguity set out in the Duchin Report and my understanding of how Wisconsin courts have previously interpreted contiguity. *See* Duchin Report at 9. Based on that

[3] The Legislature's report also measures the Schwartzberg score, which can be derived as the reciprocal of the squareroot of the Polsby-Popper score and hence does not provide any additional ranking information [Duchin and Tenner 2018].

measure, all plans contain contiguous districts. As a result, I do not report on this metric in my analysis below.

IV.F Nesting

Wisconsin also has a requirement that each Senate district be formed by merging the units associated to three contiguous Assembly districts, as "no assembly district shall be divided in the formation of a senate district." Wis. Const. art. IV, § 5; Order ¶ 37. Each of the proposed plans satisfies this requirement. As a result, I do not report on this metric in my analysis below.

IV.G Least Change

As the Duchin Report notes, the Court's November 30 order specifies that parties should "make only necessary modifications to accord with the legal requirements" and avoid "treading further than necessary to remedy [the existing maps'] currently legal deficiencies." To evaluate how the proposed plans perform on this least change principle, I apply metrics elaborated in the Duchin Report, including measures of core people retention, core area retention, buffer distance, county overlap, and district overlap. See Duchin Report at 10. I also apply another discrete approach to measuring least change, considering whether adjacent census blocks are placed in the same or different districts in the enacted plan and whether that configuration is preserved in the proposed plan. This is similar to the cut edges measure defined for compactness above. I calculated this on the block-level dual graph for all plans, so the baselines are given by the number of edges of each type reported for the enacted plan.

IV.H Traditional Districting Criteria

In the November 30 Order, the concurring opinion specifically mentions the preservation of communities of interest and minimizing the number of individuals who must wait six years between voting for a state senator as examples of 'traditional districting criteria' that might be considered to distinguish between otherwise lawful maps determined to have been drawn with a least-change approach.[4]

IV.H.1 Communities of Interest

The preservation of communities of interest is a traditional districting principle. In reviewing the Bewley, BLOC, Congressman, Governor, Hunter, Legislature, and Johnson plans, briefs, and expert reports submitted on December 15, I did not identify any significant quantitative analysis of efforts to preserve communities of interest. The parties do offer some examples of communities of interest they assert have been preserved in their plans. *See*, e.g., Congressman's Br. at 12-18; Hunter Br. at 16-17. While I offer no analysis of these examples, some parties also draw on the reduction of county, municipal, and/or ward splits as evidence that their plans protect communities of interest. *See*, e.g., Hunter Br. at 16-17. As discussed below, in Parts V.B.4, V.C.3, and V.D.2, the CMS plans perform very well on these measures.

IV.H.2 Delayed Voting in Senate Elections

While statewide elections occur in Wisconsin in each even year, only half of the State Senate districts hold elections at a time, as the Senate terms are for four years. The cycle of each district is determined by whether it is odd or even. Thus, any individuals that are moved from an odd-numbered district to an even-numbered district as a result of redistricting face a six-year gap between being able to vote for a State Senator. Minimizing the number of people placed into this situation has been used to evaluate Senate districts in past cycles. In evaluating how plans perform on this metric, I offer two computations. First, I measure what proportion of the population is currently in an odd-numbered district but would be moved to an even-numbered district under each proposal. Second, I measure the absolute number of people moved from an odd-numbered district to an even-numbered district under each proposal.

[4] As mentioned above, criteria like compactness and political boundary preservation that are required for constructing Assembly districts are also frequently referred to as traditional districting criteria when analyzing Congressional maps.

V Analysis and Methodology

This section describes the results of the analysis that was used to inform the executive summary above.

V.A Data

V.A.1 Methodology

The data used in this report is described in Appendix B. The starting point for my quantitative analysis was the LTSB block-level shapefile, which contained population values and subcategories derived from the census data release. Block equivalence files were used to associate each party's plan with the census block units, with one exception. Because the Bewley maps had some values labelled with sub-block assignments, I used the MAUP package to assign the district boundary level shapefiles to the blocks. This accounts for some small deviations in measurements obtained, as compared to those in the Bewley intervenors' expert report, particularly with respect to ward splits. In the tables below I include the values from their report parenthetically where there are significant discrepancies, particularly with respect to ward splits. In the discussion of the maps I use the most beneficial values to the Bewley plan for each metric.

For the vote total computations necessary to evaluate district performance for VRA analysis I used the ward level geographic data provided by the LTSB, and supplemented it with additional data provided by counsel for the Citizen Mathematicians and Scientists. To analyze plans not drawn on wards, I associated votes with blocks by using the MAUP package to prorate vote totals from the ward level (using the voting age populations). As there are several wards with no population, but some votes recorded, this leads to some small discrepancies in vote totals. However, aggregated back to the district level, this appears to have minimal impact on the final results.

V.A.2 Changes in Wisconsin Demography

In evaluating how the proposed maps perform on various metrics, it is important to remember why the existing map must be adjusted. During the past decade, the population of Wisconsin has grown by 3.6%. However, that population growth is not equally distributed across the state. For example, Dane County grew by over 15%, but nearly 30% (21/72) of counties have lost population since 2010. The last decade has also seen other shifts in demography and in the boundaries of political subdivisions. Changes in the relative density of minority groups throughout the state have not been uniform and several town and city boundaries have been modified since the 2011 maps were enacted. *See* Citizen Mathematicians and Scientists Brief at 5. These shifts can affect the line-drawing process and performance on various redistricting requirements and criteria.

V.B Congressional Maps

In this section I analyze the three proposed alternatives to the CMS congressional map, based on the metrics and principles described above. In all tables, I italicize those metrics for which a lower score is better, and leave unitalicized those metrics for which a higher score is better.

V.B.1 Population Deviation

As discussed above, Congressional district populations should be zero-balanced. Thus, in an optimal plan, I expect the difference between the largest district population and the smallest district population to be exactly one person.

In Table 1,[5] I report population deviation for the proposed plans and the plan enacted in 2011. The Table demonstrates three important points. First, the enacted plan is significantly malapportioned. This underscores the magnitude of reapportionment required. Second, the Legislature and CMS reduce the range

[5]Here and throughout, if a criterion must be considered under applicable law (as I understand it) then I highlight the best performing map on metrics evaluating that criterion in green.

of population deviation to 1. Third, the Governor and Hunter plans do not achieve that range of population deviation, because neither reduces the difference between the most and least populous districts to a single person. In this respect, the table demonstrates that it is important to compare the size of proposed districts when calculating the range of population deviation, rather than comparing districts to the rounded ideal population, *see* Governor's Brief at 12, or evaluating population based on a plus or minus calculation, *see* Hunter Brief at 15. Ultimately the key takeaway is that the Legislature and CMS plans achieve the mathematically optimal population deviation, and the Governor and Hunter plans do not.

Plan Name	CMS	GOV	HUNTER	SB622	WI-ENACTED
Maximum Deviation	0	1	1	0	52,681
Minimum Deviation	-1	-1	-1	-1	-41,320
Min to Max	1	2	2	1	94,001

Table 1: Population deviations for proposed Congressional maps.

V.B.2 Voting Rights Act

The parties broadly agree on the impact of the VRA on the Congressional map, with each plan supporting a single minority opportunity district that is majority non-White as shown in Table 2.

Plan Name	CMS	GOV	HUNTER	SB622	WI-ENACTED
Minority Opportunity	1	1	1	1	1

Table 2: VRA summary of proposed Congressional districts

V.B.3 Least Change

In Table 3, I report several metrics of least change. First, core population and population percent (together "core population retention") reflect the number of people that are moved to a new district in the proposed plan. Second, core area moved and area percent (together "core area retention") reflect the amount of area that is moved to a new district in the proposed plan. Third, county and district overlap reflect whether at least one county and populated unit (respectively) are preserved between identically numbered districts in the enacted and proposed map. Fourth, preserved internal edges reflects the number of adjacent census blocks that are preserved between identically numbered districts. Fifth, buffer distance reflects the amount of buffering that must be done in order to contain all of an enacted district in the new map. These metrics represent just a few of the metrics that can be used to evaluate whether a map comports with the least change approach established in the Court's November 30 Order.

Core population retention and core area retention are familiar metrics that provide a rough proxy for the magnitude of changes made to the enacted map. As Professor Duchin explained in her initial report, the county and district overlap figures are grounded in the Court's November 30 opinion and, more specifically, the concurring opinion. Preserved internal edges is a more discrete computation of change. The preserved internal edges calculation accounts for the number of census block adjacencies (i.e. a discrete unit of area) that are preserved in the new plan, reflecting a local measurement of the nearest pairs of residents that can be separated in a districting plan.

Given the magnitude of malapportionment that must be remedied, I expect that all parties will need to make material adjustments to the enacted map. I am not aware of any quantitative threshold that should be applied in evaluating how parties perform on these continuous-valued metrics of least change. Table 3 illustrates that all parties perform well on at least some metrics, and that there is variance in how well most parties perform. For example, the Legislature moves the second-least number of people, but the most area. The Governor performs best on both core people retention and core area retention, but does not perform best on preserved internal edges, on which the Hunter map performs best. This illustrates that various approaches to making minimal changes to the starting plan may lead measurably different outcomes. All parties score perfectly on both county and district overlap. I do not claim that these metrics exhaust the mechanisms for evaluating whether a map comports with the least change approach.

Plan Name	CMS	GOV	HUNTER	SB622
Core Population Moved	500785	324415	411777	384456
Population Percent Moved	8.5	5.5	7.0	6.5
Area Percent Moved	3.0	1.5	3.4	9.1
Average Buffer Distance	5.1	4.8	9.6	11.5
County Overlap	8	8	8	8
District Overlap	8	8	8	8
Preserved Internal Edges	487096	487087	487245	486746

Table 3: Least change metrics for proposed Congressional maps

V.B.4 Political Boundaries

In Table 4, I report metrics that demonstrate the degree to which parties preserve political boundaries, including counties, municipalities,[6] and wards. The measurement of municipality splits presented here agrees with the approach taken by Professor Duchin in her report. I understand these metrics to be appropriate considerations in selecting between plans that comply with federal legal requirements and comport with the least change approach.

Given the size of Congressional districts, and the challenges of zero-balancing, I expect that all plans will split some number of wards and counties. Specifically based on my experience analyzing Congressional redistricting maps, I would expect that in a strong map the number of county splits would be approximately on the order of the number of districts or smaller. And I expect the number of municipal splits to be larger than the number of county splits, because municipal boundaries do not align directly with county boundaries, because municipalities are more numerous than counties, and because I understand that prioritizing the preservation of county lines is consistent with the Wisconsin constitution and historical practice.

I am not aware of any quantitative threshold that should be applied in evaluating how parties perform on continuous-valued metrics of boundary preservation. Among alternatives to the CMS plan, the Hunter plaintiffs perform best on wards and municipalities, while performing nearly as well as the Legislature on county splits and just as well on county pieces. However, Table 4 illustrates that the CMS maps are comfortably best on all metrics, splitting many fewer wards, fewer counties, and fewer municipalities than the second best map, while also creating the fewest pieces.

Each plan splits at least one county that is not split in the others. For example, the CMS map is the only proposal that splits Iowa county. The counties not split by the CMS map but divided by the other proposals are Jackson, Juneau, Monroe, Sauk, Walworth, and Winnebago for the Governor's map; Monroe, Sauk, Shawano, Winnebago, and Winnebago for the Hunter map; and Columbia, Dunn, Portage, Sauk, Walworth, and Waukesha for the Legislature's map.

Similarly, with respect to municpalities, the five largest split by each plan that is not split in the CMS plan are West Allis, Tomah, East Troy, Lomira, and Eagle Point in the Governor's plan; Wauwatosa,

[6] My measure of municipalities includes both unincorporated towns and incorporated cities and villages. In the state legislative context, I understand that respect for certain municipalities (incorporated cities and villages) is not required under the Wisconsin Constitution. However, I also understand that Wisconsin courts have (historically) evaluated the number of cities and villages, as well as the number of towns, that are split. As a result, I included cities and villages in my analysis of the congressional and legislative maps.

Waupun, Tomah, East Troy, and Columbus in the Hunter Plan; and Waukesha, Eau Claire, Wauwatosa, New Berlin, and Menomonee Falls in the Legislature's plan.

Plan Name	CMS	GOV	HUNTER	SB622	WI-ENACTED
Split Wards	8	32	18	48	0
Split Counties	7	12	11	10	12
County Pieces	15	25	22	22	27
Pop Split Counties	7	12	10	10	12
Pop Split County Pieces	7	12	10	11	14
Split Munis	13	30	20	24	35
Muni Pieces	27	60	40	48	70
Pop Split Munis	13	30	20	24	35
Pop Split Muni Pieces	14	30	20	24	35

Table 4: Political boundary preservation in proposed Congressional maps.

V.B.5 Compactness

In Table 5, I report metrics that demonstrate the degree to which parties achieve compactness in their Congressional districts. As with respect to political boundaries, I understand these metrics to be appropriate considerations in selecting between plans that comply with federal legal requirements and comport with the least change approach.

As with several prior metrics, I am not aware of any quantitative threshold that should be applied in evaluating how parties perform on these continuous-valued metrics. However, I understand that in the state legislative context, compactness should be maximized to the extent practicable, and that for Congressional districts this is often viewed as a traditional districting criterion.

Among alternatives to the CMS plan, the Legislature's plan appears to perform better than the Governor or Hunter plans, achieving a higher mean Polsby-Popper Score, a higher mean convex hull score, and a better cut edges score.[7]

However, on metrics of compactness, the Legislature's plan does not outperform the CMS plan. As Table 5 illustrates, the CMS plan performs best in all mean categories except for convex hull, where it performs just .003 points behind the Legislature. In my view, it is particularly noteworthy that the CMS plan performs best on the cut edges metric, as that metric controls for natural features, like the Wisconsin coastline

Plan Name	CMS	GOV	HUNTER	SB622	WI-ENACTED
Min Polsby-Popper	0.126	0.127	0.125	0.125	0.118
Mean Polsby-Popper	0.305	0.243	0.272	0.280	0.209
Min Reock	0.360	0.334	0.286	0.334	0.302
Mean Reock	0.464	0.458	0.425	0.456	0.440
Min Convex Hull Ratio	0.638	0.592	0.590	0.679	0.588
Mean Convex Hull Ratio	0.776	0.758	0.733	0.779	0.741
Cut Edges	3228	3774	3661	3410	4293

Table 5: Compactness performance of Congressional district proposals

[7] Unlike other metrics of compactness, cut edges is a plan-wide rather than district-specific score. With respect to cut edges, a lower figure demonstrates greater compactness.

V.B.6 Congressional Summary

Among the alternatives to the CMS congressional map that I was instructed to analyze, and applying my understanding of the law applicable to Wisconsin congressional redistricting, I conclude that the Legislature's map performs better than other alternatives to the CMS plan on federal requirements, that the Governor's map performs better than other alternatives to the CMS plan on metrics of least change, and that each party performs better than other alternatives to the CMS plan on at least one criterion that is considered traditional and neutral when evaluating congressional maps. However, I further conclude that the CMS performs just as well as the Legislature's map on federal requirements, outperforms or equals the Governor on some metrics of least change, and performs best on each of the traditional neutral criteria considered in this report. Accordingly, after reviewing and analyzing alternatives to the CMS plan, I am not able to identify any that performs as effectively under the applicable framework.

V.C State Senate

Next, I turn to analyzing the five proposed alternatives to the CMS Senate map, based on the metrics and principles described in Part IV of my report.

V.C.1 Population Deviation

The Court's November 30 order instructs that, for legislative district populations, "there should be as close an approximation to exactness as possible." Order ¶ 33. Although the Court did not identify a specific quantitative threshold, I expect that in preparing proposed plans parties will seek to minimize top to bottom deviations from the ideal district size.

In Table 6 I report the population deviation values for the proposed and current plans. The Table illustrates three important points. First, as with Congressional districts, the malapportionment of the current plan is easy to observe and underscores the magnitude of reapportionment required. Second, it is possible to drive population deviation down well below the top-to-bottom deviations in the Bewley, BLOC, Governor's, and Hunter plans. Third, the CMS plan performs better than all alternative plans on each measure of deviation. In this respect, the CMS proposal stands out. While the Legislature's population deviation figures may be "remarkable" [Legislature's Br. at 8] and "exceptionally good" *see* Bryan Report ¶ ¶ 45, 49, the CMS plan performs better on this critical criterion.

Plan Name	BEWLEY	BLOC	CMS	GOV	HUNTER	SB621	WI-ENACTED
Maximum Deviation	1281	802	428	1112	845	520	22874
Maximum Deviation (%)	0.718	0.449	0.24	0.623	0.473	0.291	12.808
Minimum Deviation	–1590	-917	-467	-1026	-853	-506	-16529
Minimum Deviation (%)	–0.89	-0.513	-0.261	-0.574	-0.477	-0.283	-9.255
Min to Max	2871	1719	895	2138	1698	1026	39403
Min to Max (%)	1.608	0.962	0.501	1.197	0.951	0.574	22.062

Table 6: Population deviations for proposed Senate maps

V.C.2 Voting Rights Act

As with the Congressional plans, there is broad agreement between the briefs, reports, and proposals on the feasibility and desirability of having two minority opportunity districts at the State Senate level. There is a small amount of variance between the plans in terms of whether they construct Black voting-age population ("BVAP")-majority districts or are content with effective districts and analyzing non-White or majority-minority districts as described in the Governor and Hunter briefs respectively.

Plan Name	BEWLEY	BLOC	CMS	GOV	HUNTER	SB621	WI-ENACTED
BVAP Opportunity	2	2	2	2	2	2	2

Table 7: VRA performance of proposed Senate maps

V.C.3 Political Boundaries

While minimizing population deviation is the first and most critical concern in redistricting, I am instructed that the Wisconsin Constitution requires respect for counties, towns, and wards when reapportioning legislative districts. See Wis. Const. art. IV, §4.[8] As a result, I consider this criterion before evaluating how the plans perform on metrics of least change.

In evaluating whether plans effectively protect political boundaries, I am cognizant of the tension between preserving counties, on the one hand, and equalizing population, on the other. That tension is exacerbated in the legislative context, because Senate and Assembly districts are smaller than congressional districts and therefore less suited to the preservation of county units with considerable population. As a result of these issues, I expect that parties able to drive population deviation closer to exactness will perform worse on metrics that concern respect for counties.[9] Because wards are a smaller unit, with less population, I anticipate diminished need to split wards in pursuit of population balance.

In Table 8, I report metrics that demonstrate the degree to which parties preserve political boundaries, including counties, municipalities, and wards. I am not aware of any quantitative threshold that should be applied in evaluating how parties perform on continuous-valued metrics of preservation. However, consistent with the requirement that plans respect counties, towns, and wards, I conclude that parties perform better on metrics of preservation when they minimize the number of splits. And Table 8 illustrates a very important point: that it is possible to navigate the tension between population balancing and respect for political boundaries.

In this respect, this analysis upends my expectation that parties performing best on population deviation will perform worse on metrics of preservation. Among alternatives to the CMS plan, the Legislature's map performed better on population deviation and performs better on metrics of preservation, preserving at least as many counties and wards, and more municipalities than other alternatives to the CMS map. As Table 8 makes clear, the BLOC, Governor, and Hunter plans split a considerable number of wards, while the Legislature's and Bewley plans split none. In addition to splitting a significant number of wards, the BLOC, Governor, and Hunter plans also split wards into a significant number of pieces. A similar dynamic plays out with respect to municipalities, where the Legislature's plan beats the other plans by a significant margin.

Although the Legislature's plan outperforms the Bewley, BLOC, Governor, and Hunter plans, it does **not** outperform the CMS plan. Although the CMS plan performs best on each metric of population deviation, and one might expect performance on preservation to suffer by consequence, the CMS plan is comfortably best on the preservation of counties, splitting 14 fewer units into 29 fewer pieces than the Legislature's plan. The CMS plan also performs just as well as the Legislature on preserving wards, splitting none. And the CMS map performs nearly as well as the Legislature on preservation of municipalities, a metric that I give less weight because I am informed that preservation of city and village lines is desirable but not mandatory under applicable law. Ultimately, based on my review and analysis of these preservation metrics, and my understanding of applicable law, I conclude that the CMS performs best on the preservation criterion.

The five largest counties split by each plan that are not split in the CMS plan are Winnebago, Kenosha, La Crosse, Dodge, and Portage for Bewley; Winnebago, Kenosha, St. Croix, Dodge, and Chippewa for

[8] Although this provision applies to Assembly districts, those Assembly districts must be nested into Senate districts, which has the practical effect of extending the requirement from Assembly to Senate districts.

[9] For the same reason, I expect that parties able to drive population deviation closer to exactness will perform worse on metrics that concern respect for municipalities. I address the inclusion of municipalities below.

Plan Name	BEWLEY	BLOC	CMS	GOV	HUNTER	SB621	WI-ENACTED
Split Wards	161 (0)	65	0	179	132	0	0
Ward Pieces	326 (0)	130	0	362	271	0	0
Pop Split Wards	165 (0)	65	0	183	139	0	0
Split Counties	48	42	28	45	42	42	46
County Pieces	138	128	86	136	116	115	130
Pop Split Counties	48	42	27	45	42	42	46
Pop Split County Pieces	77	73	45	78	61	60	71
Split Muni	67 (52)	73	31	117	109	28	84
Muni Pieces	146	157	69	252	234	62	180
Pop Split Munis	67	73	31	117	109	28	84
Pop Split Muni Pieces	75	80	34	131	121	30	92

Table 8: Political boundary preservation in proposed Senate maps.

BLOC; Winnebago, Kenosha, St. Croix, Dodge, and Portage for the Governor; Winnebago, Kenosha, La Crosse, Dodge, and Portage for Hunter; and Winnebago, Kenosha, La Crosse, St. Croix, and Dodge for the Legislature.

The five largest municipalities split by each plan that are not split in the CMS proposal are Racine, West Allis, Beloit, Menomonee Falls, and Middleton for the Bewley proposal; Kenosha, Racine, Oshkosh, Wesst Allis, and Sheboygan for the BLOC proposal; Kenosha, Racine, West Allis, Beloit, and Menomonee Falls for the Governor's proposal; Kenosha, Racine, West Allis, Menononee Falls, and Manitowoc for the Hunter proposal; and Racine, West Allis, Beloit, Waterford, and Tomah for the Legislature's proposal.

V.C.4 Compactness

Although it bears repeating that minimizing population deviation is the first and most critical concern in redistricting, I am instructed that the Wisconsin Constitution requires that plans create legislatively districts "in as compact form as practicable." Wis. Const. art. IV, § 4. As a result, and as with respect for political boundaries, I consider this criterion before evaluating how the parties' plans also perform on metrics of least change. The results of my analysis are reported in Table 9. Although I am not aware of any quantitative threshold that should be applied in evaluating how parties perform on these continuous-valued metrics, the requirement to make districts in "as compact form as practicable" suggests parties should attempt to maximize the compactness of districts, subject to the constraints created by other redistricting requirements.

In evaluating alternatives to the CMS plan, I determine that the Hunter plan performs better than other plans on metrics of compactness, as it is the only plan to increase the prevailing mean Polsby-Popper and convex hull ratio scores, and also performs best on cut edges. However, as Table 9 illustrates, the CMS map performs very competitively on measures of compactness, achieving the best mean Reock score and placing second to the Hunter plan on mean Polsby-Popper, mean Convex-Hull, and cut edges. Significantly, the CMS achieves this performance while splitting significantly fewer political boundaries than the Hunter plan. I say this is significant because superior compliance with the requirement to respect county, town, and ward boundaries necessarily constrains the ability to improve compactness. Despite the constraints imposed by its superior compliance on another redistricting requirement, the CMS plan manages to achieve districts that are nearly as or as compact as those in the Hunter plan.[10]

[10]Of course, the CMS plan also performs better on mean metrics of compactness than every alternative to the Hunter plan, including the only other plan that is competitive on metrics of preservation: the Legislature's.

Plan Name	BEWLEY	BLOC	CMS	GOV	HUNTER	SB621	WI-ENACTED
Min Polsby-Popper	0.078	0.067	0.071	0.053	0.085	0.048	0.053
Mean Polsby-Popper	0.213	0.197	0.260	0.217	0.268	0.224	0.230
Min Reock	0.137	0.127	0.140	0.135	0.141	0.133	0.128
Mean Reock	0.401	0.395	0.402	0.392	0.397	0.395	0.402
Min Convex Hull Ratio	0.492	0.486	0.508	0.480	0.470	0.466	0.483
Mean Convex Hull Ratio	0.717	0.695	0.735	0.710	0.739	0.710	0.723
Cut Edges	10688	11776	9754	11147	9565	10785	10928

Table 9: Compactness in proposed Senate maps.

V.C.5 Least Change

Legislative plans must comply with federal and state constitutional requirements, including requirements concerning population deviation, compliance with the VRA, respect for county, town, and ward lines, and maximizing compactness (to the extent practicable). My analysis of metrics relevant to each requirement—including metrics that illustrate whether plans approximate population balance as closely as possible, contain a sufficient number of districts that can elect minority candidates of choice, respect counties and wards, and maximize compactness to the extent practicable—demonstrates that the alternatives to the CMS plan fall short. Nevertheless, I have been asked to evaluate how the parties' plans perform on metrics of least change. The results of that evaluation are reported in Table 10.

In evaluating performance on these measures of least change, it is important to remember the magnitude of reapportionment required to approximate population balance as closely as possible, and to consider the effect that compliance with requirements concerning the integrity of counties, towns, and wards, as well as compactness, have on the ability to minimize change. Because of the requirements applicable to legislative redistricting in Wisconsin, I expect that parties will need to tolerate more change in order to "conform the existing districts to constitutional and statutory requirements." Order ¶8. Moreover, the smaller size of Senate districts make them more sensitive to local population shifts, including those made to preserve political boundaries.

Among alternatives to the CMS map, the Legislature's plan prevails on core population retention, with the Governor's plan prevailing on core area retention, buffer distance, and preserved internal edges. While the CMS plan does not perform as well as others on the core retention metrics or buffer distance, it performs well on metrics of county and district overlap and preserved internal edges. Moreover, the CMS plan's performance on least change is not surprising, given that it performs best on the requirement to balance population, best on the requirement to preserve county, town, and ward boundaries, and nearly best on the requirement to maximize compactness to the extent practicable. In this respect, the metrics of least change reflect the trade-offs necessarily made in pursuit of constitutional and statutory requirements.

Plan Name	BEWLEY	BLOC	CMS	GOV	HUNTER	SB621
Core Population Moved	576321	610568	1513824	461228	1128878	459061
Percent Population Moved	9.8 (9.5)	10.4	25.7	7.8	19.2	7.8
Percent Area Moved	9.8	6.1	29.1	5.0	14.0	7.1
Average Buffer Distance	6.7	6.2	17.0	5.4	8.5	6.5
County Overlap	33	33	33	33	33	33
District Overlap	33	33	33	33	33	33
Preserved Internal Edges	476575	476621	477230	477745	476482	477558

Table 10: Least change metrics of proposed Senate maps

V.C.6 Delayed Voting in Senate Elections

Counsel also asked that I evaluate the extent to which proposed maps result in delayed voting for senators. The result of my analysis is reported below, in Table 11.

Plan Name	BEWLEY	BLOC	CMS	GOV	HUNTER	SB621
Odd to Even Population	137084	177698	422492	139677	240593	138753
Percent	2.3	3	7.2	2.4	4.1	2.4

Table 11: Count of individuals with a six-year delay in Senate votes.

V.C.7 State Senate Summary

Among alternatives to the CMS plan, there is significant variance in performance across metrics related to constitutional and statutory requirements, and on metrics of least change. Having analyzed the performance of each plan across metrics related to constitutional and statutory requirements, I conclude that the plans fall short of the CMS plan, which necessarily makes modifications to district boundaries to achieve lower population deviation, fewer county splits, and better mean Reock scores than any other plan, while also performing as well as other plans on metrics related to VRA compliance and the preservation of wards, and performing better than nearly all maps on several important measures of compactness, including mean Polsby-Popper, mean Convex Hull, and cut edges.

V.D State Assembly

I conclude by analyzing the five alternatives to the CMS Assembly map, again based on the metrics and principles set out in Part IV of my report.

V.D.1 Population Deviation

As with respect to Senate districts, and consistent with the requirements that plans approximate exactness as closely as possible, I expect that parties will aim to minimize top-to-bottom population deviation from the ideal district size. In Table 12, I report the population deviation values for the proposed and current plan. The results are similar to those obtained for Senate districts and support three conclusions. First, the malapportionment of the current plan is significant and illustrates the magnitude of reapportionment required. Second, population deviation can be driven down well below levels achieved by the Bewley, BLOC, Governor, and Hunter plans. Finally, the Legislature and CMS plans again achieve the tightest balance, with the CMS plan obtaining even more “remarkable” proximity to exactness.

Plan Name	BEWLEY	BLOC	CMS	GOV	HUNTER	SB621	WI-ENACTED
Maximum Deviation	547	392	220	584	530	231	12183
Maximum Deviation (%)	0.92	0.659	0.37	0.982	0.891	0.389	20.465
Minimum Deviation	-557	-392	-218	-537	-553	-221	-6905
Minimum Deviation (%)	-0.935	-0.658	-0.365	-0.901	-0.928	-0.37	-11.598
Min to Max	1104	784	438	1121	1083	452	19088
Min to Max (%)	1.854	1.317	0.736	1.883	1.819	0.759	32.063

Table 12: Population deviations for proposed Assembly maps

V.D.2 Voting Rights Act

There is a little more of interest with respect to the VRA at the level of Assembly districts. Several of the proposals are able to construct an additional effective opportunity district compared to the 2011 plan, although as with the Senate districts there is some variance between the formulations. In particular, the

BLOC plaintiffs construct 7 majority BVAP districts, while the CMS map contains 7 effective districts at a wider range of BVAP percentages. With respect to HVAP the proposals are consistent, providing two majority HVAP districts.

Plan Name	BEWLEY	BLOC	CMS	GOV	HUNTER	SB621	WI-ENACTED
BVAP Opportunity	6	7	7	7	7	6	6
HVAP Opportunity	2	2	2	2	2	2	2

Table 13: VRA performance for proposed Assembly maps

V.D.3 Political Boundaries

As with respect to Senate districts, and consistent with the requirement that plans respect counties, towns, and wards when reapportioning legislative districts, I view plans favorably if they minimize the number of splits. And as with respect to Senate districts, the proposals demonstrate that it is possible to navigate the tension between minimizing population deviation and respect for political boundaries.

Table 14 demonstrates that, among alternatives to the CMS plan, there is no clear winner on preservation of political boundaries. However, I conclude that the CMS plan outperforms each alternative plan. Like the Legislature and Bewley plans, the CMS plan preserves every single ward. And, importantly, the CMS plan splits the fewest number of counties into the fewest pieces Although the CMS map splits more municipalities into more pieces than does the Legislature, I weight those splits less than county splits, because I am informed that preservation of city and village lines is desirable but not mandatory under applicable law.

Plan Name	BEWLEY	BLOC	CMS	GOV	HUNTER	SB621	WI-ENACTED
Split Wards	285 (0)	94	0	258	257	0	0
Ward Pieces	583 (0)	190	0	529	527	0	0
Pop Split Ward Pieces	298 (0)	96	0	271	270	0	0
Split Counties	55	53	40	53	50	53	58
County Pieces	229	226	175	229	194	212	229
Pop Split Counties	54	52	33	52	48	51	57
Pop Split County Pieces	109	108	70	111	79	94	106
Split Munis	99 (79)	104	70	175	181	48	126
Muni Pieces	250	254	176	415	421	125	296
Pop Split Munis	97	101	66	174	180	45	125
Pop Split Muni Pieces	129	128	84	218	218	55	148

Table 14: Political boundary preservation in proposed Assembly maps

V.D.4 Compactness

As with respect to Senate districts, and consistent with the Wisconsin constitutional requirement that parties create Assembly districts "in as compact form as practicable," I expect that plans will attempt to maximize the compactness of districts, subject to the constraints created by other districting requirements. Among alternatives to the CMS plan, the Hunter plan performs best on all mean measures of performance and cut edges. The CMS plan performs competitively on these measures, placing second to the Hunter plan on each one, and accomplishing this notwithstanding materially greater respect for counties, municipalities, and wards. Particularly given that the required pursuit of compactness is subject to a practicability requirement, and that respect for political subdivisions necessarily trades off against the flexibility need to maximize compactness, I conclude that the CMS plans perform very well on this measure.

Plan Name	BEWLEY	BLOC	CMS	GOV	HUNTER	SB621	WI-ENACTED
Min Polsby-Popper	0.065	0.043	0.062	0.056	0.086	0.050	0.048
Mean Polsby-Popper	0.253	0.227	0.282	0.251	0.340	0.243	0.260
Min Reock	0.148	0.136	0.148	0.147	0.136	0.148	0.147
Mean Reock	0.405	0.374	0.406	0.397	0.442	0.379	0.390
Min Convex Hull Ratio	0.473	0.351	0.473	0.475	0.449	0.291	0.418
Mean Convex Hull Ratio	0.734	0.698	0.736	0.720	0.783	0.717	0.732
Cut Edges	18420	20096	17781	18441	15353	19196	18994

Table 15: Compactness metrics for proposed Assembly maps

V.D.5 Least Change

As with respect to Senate districts, it bears repeating that Assembly districts must comply with federal and state constitutional requirements, including requirements concerning population deviation, compliance with the VRA, respect for counties, towns, and wards, and maximizing compactness (to the extent practicable). My analysis of metrics relevant to each requirement demonstrates that the alterna-tives to the CMS plan fall short. Nevertheless, I have been asked to evaluate how the parties' plans perform on metrics of least change.

The results of that evaluation are reported in Table 16, and I again note that in evaluating performance on these measures of least change, it is important to remember the magnitude of the reapportionment required to approximate population balance as closely as possible, and to consider the effect of compliance with other constitutional and statutory requirements. Under the circumstances, and given that Assembly districts are particularly sensitive to local population shifts, it is not surprising that the CMS plan does not perform as well as others on the reported metrics. The results provide further evidence that metrics of least change reflect trade-offs necessarily made in pursuing constitutional and statutory requirements.

Plan Name	BEWLEY	BLOC	CMS	GOV	HUNTER	SB621
Core Population Moved	984336	939513	2299625	837659	1586059	933604
Population Percent Moved	16.7 (16.2)	15.9	39.0	14.2	26.9	15.8
Area Percent Moved	16.8	9.6	38.5	11.3	18.2	16.5
Average Buffer Distance	5.4	4.9	13.0	4.8	6.0	6.0
County Overlap	99	99	93	99	99	99
District Overlap	98	99	85	99	99	99
Preserved Internal Edges	465157	466205	465050	467562	466597	466249

Table 16: Least change analysis for proposed Assembly maps

V.D.6 Assembly Summary

Among alternatives to the CMS plan, I again find considerable variance in performance across metrics related to constitutional and statutory requirements, and on metrics of least change. Based on my analysis of each plan across metrics related to those constitutional and statutory requirements, I conclude that the plans again fall short of the CMS plan, which necessarily makes modifications to district boundaries to achieve the lowest population deviation, the fewest county splits, and the second best score on critical measures of compactness, including mean Reock, mean Polsby-Popper, mean Convex Hull, and cut edges. The CMS plan accomplishes all of that without splitting a single ward, and while performing effectively on metrics related to VRA compliance.

VI References

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[Solomon and Barnes 2021] Richard Barnes and Justin Solomon, Gerrymandering and Compactness: Implementation Flexibility and Abuse. Political Analysis, Volume 29, Issue 4, 2021, 448–466.

[Duchin and Tenner 2018] Moon Duchin and Bridget Tenner, Discrete geometry for electoral geography. ArXiv: 1808.05860, 2018, 1-18.

I declare under penalty of perjury of the United States that the foregoing is true and correct to the best of my knowledge and understanding.

Dated: December 30, 2021

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Daryl R. Deford

Appendix A

DARYL R. DEFORD

Curriculum Vitae

328 Neill Hall WSU Pullman, WA ⋄ (509) 205–7347

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ACADEMIC APPOINTMENTS

Washington State University, Pullman, WA *August 2020 – Present*
Assistant Professor of Data Analytics – Department of Mathematics and Statistics

Massachusetts Institute of Technology, Cambridge, MA *June 2018 – July 2020*
Postdoctoral Associate – CSAIL Geometric Data Processing Group
Advisor: Justin Solomon

Tufts University, Medford, MA *June 2018 – July 2020*
Visiting Scholar – Jonathan M. Tisch College of Civic Life
Advisor: Moon Duchin

EDUCATION

Dartmouth College, Hanover, NH *September 2013 – June 2018*
Ph.D. Mathematics *Awarded June 2018*
Advisor: Dan Rockmore
Dissertation: Matched Products and Dynamical Models for Multiplex Networks
A.M. Mathematics *Awarded November 2014*

Washington State University, Pullman, WA *August 2010 – May 2013*
B.S. in Theoretical Mathematics *Awarded May 2013*
Summa Cum Laude

RESEARCH PUBLICATIONS

Accepted Papers

A24: *Random Walks and the Universe of Districting Plans* (with M. Duchin), Book Chapter in *Political Geography*, Birkhäuser, to appear 2022.

A23: *Implementing Partisan Symmetry: Problems and Paradoxes* (with N. Dhamankar, M. Duchin, V. Gupta, M. McPike, G. Schoenbach, K. W. Sim), Political Analysis, arxiv: 2008.06930, to appear 2022.

A22: *Empirical Sampling of Connected Graph Partitions for Redistricting* (with L. Najt and J. Solomon), Physical Review E, 104(6), 064130, 2021.

A21: *Partisan Dislocation: A Precinct-Level Measure of Representation and Gerrymandering* (with N. Eubank and J. Rodden), Political Analysis, 1-23, doi:10.1017/pan.2021.13, 2021.

A20: *Colorado in Context: Congressional Redistricting and Competing Fairness Criteria in Colorado* (with J. Clelland, H. Colgate, B. Malmskog, and F. Sancier-Barbosa), Journal of Computational Social Science, doi:10.1007/s42001-021-00119-7, 2021.

A19: *ReCombination: A family of Markov chains for redistricting* (with M. Duchin and J. Solomon), Harvard Data Science Review, 3(1), 2021.

A18: *Medial Axis Isoperimetric Profiles* (with J. Solomon and P. Zhang), Computer Graphics Forum, 39(5), 1-13, 2020.

A17: *On the Spectrum of Finite, Rooted Homogeneous Trees* (with D. Rockmore), Linear Algebra and its Applications, 598, 165-185, 2020.

A16: *Competitiveness Measures for Evaluating Districting Plans* (with M. Duchin and J. Solomon), Statistics and Public Policy, 7(1), 69-86, 2020.

A15: *Mathematics of Nested Districts: The Case of Alaska* (with S. Caldera, M. Duchin, S. Gutenkust, and C. Nix), Statistics and Public Policy, 7(1), 39-51, 2020.

A14: *Aftermath: The ensemble approach to political redistricting* (with J. Clelland and M. Duchin), MAA Math Horizons, 28(1), 34-35, 2020.

A13: *Total Variation Isoperimetric Profiles* (with H. Lavenant, Z. Schutzman, and J. Solomon), SIAM J. Appl. Algebra Geometry, 3(4), 585-613, 2019.

A12: *Spectral Clustering Methods for Multiplex Networks* (with S. Pauls) Physica A: Statistical Mechanics and its Applications, 533, 121949, 2019.

A11: *Redistricting Reform in Virginia: Districting Criteria in Context* (with M. Duchin), Virginia Policy Review, 12(2), 120-146, 2019.

A10: *A New Framework for Dynamical Models on Multiplex Networks* (with S. Pauls), Journal of Complex Networks, 6(3), 353-381, 2018.

A9: *Cyclic Groups with the same Hodge Series*, (with P. Doyle), Revista de la Uniòn Matemática Argentina, 59(2), 241–254, 2018.

A8: *Multiplex Dynamics on the World Trade Web*, Proc. 6th International Conference on Complex Networks and Applications, Studies in Computational Intelligence, Springer, 1111–1123, 2018.

A7: *Random Walk Null Models for Time Series Data*, (with K. Moore), Entropy, 19(11), 615, 2017.

A6: *Enumerating Tilings of Rectangles By Squares*, Journal of Combinatorics, 6(3), 339-351, 2015.

A5: *Enumerating Distinct Chessboard Tilings*, Fibonacci Quarterly, 52(5), 102-116, 2014.

A4: *Pulsated Fibonacci Sequences* (with K. Atanassov and A. Shannon), Fibonacci Quarterly, 52(5), 22-27, 2014.

A3: *Seating Rearrangements on Arbitrary Graphs*, Involve: A Journal of Mathematics, 7(6), 787-805, 2014.

A2: *Empirical Analysis of Space-Filling Curves for Scientific Computing Applications* (With A. Kalyanaraman), Proc. 42nd International Conference on Parallel Processing, 170-179, 2013.

A1: *Counting Rearrangements on Generalized Wheel Graphs*, Fibonacci Quarterly, 51(3), 259-273, 2013.

Preprints

P4: *Bayesian Inference of Random Dot Product Graphs via Conic Programming* (with D. Wu and D. Palmer), arXiv:2101.02180.

P3: *Complexity and Geometry of Sampling Connected Graph Partitions* (with L. Najt and J. Solomon), arXiv: 1908.08881.

P2: *Fourier Transforms on $SL_2(\mathbb{Z}/p^n\mathbb{Z})$ and Related Numerical Experiments* (with B. Breen, J. Linehan, and D. Rockmore), arXiv:1710.02687.

P1: *A Random Dot Product Model for Weighted Networks* (with D. Rockmore) arXiv: 1611.02530.

Technical Reports

T6: *Ensemble Analysis for 2021 Legislative Redistricting in Colorado, First and Second Staff Plans* (with J. Clelland, B. Malmskog, and F. Sancier-Barbosa), Colorado in Context Report, 2021.

T5: *Ensemble Analysis for 2021 Congressional Redistricting in Colorado* (with J. Clelland, B. Malmskog, and F. Sancier-Barbosa), Colorado in Context Report, 2021.

T4: *Comparison of Districting Plans for the Virginia House of Delegates* (with M. Duchin and J. Solomon), MGGG Technical Report, 2019.

T3: *Amicus Brief of Mathematicians, Law Professors, and Students* (with M. Duchin and G. Charles et al.), Rucho v. Common Cause, Supreme Court, 2019.

T2: *Study of Reform Proposals for Chicago City Council* (with M. Duchin et al.), MGGG Technical Report, 2019.

T1: *An Application of the Permanent–Determinant Method: Computing the Z-Index of Arbitrary Trees*, WSU Department of Mathematics Technical Report Series 2013 #2, 2013.

TEACHING EXPERIENCE

Washington State University Pullman, WA
Instructor *Fall 2020 - Present*

- Designed syllabi and daily lectures. Wrote and graded homework, quizzes, and exams. Fully responsible for course content and material.

Math 448/548 - Numerical Analysis *Spring 2022*

Fundamental course on numerical computation, including: finding zeroes of functions, approximation and interpolation, numerical integration, numerical solution of ordinary differential equations, and numerical linear algebra.

STAT 419 - Introduction to Multivariate Statistics *Fall 2021*

Introductory course covering multidimensional data, multivariate normal distribution, principal components, factor analysis, clustering, and discriminant analysis.

Data 115 - Introduction to Data Analytics *Fall 2020, 2021 Spring 2021*

Basic techniques and methodology of data science, with an emphasis on data processing and software tools. This course provides a foundation for beginning data analytics majors as well as students from across the university who are looking to develop data and quantitative literacy.

Math 581 - Topics in Math (Computational Methods in Complex Networks) *Fall 2020*

Introduction to computational methods and software for analyzing complex systems as well as applications of partition sampling to political redistricting.

Metric Geometry and Gerrymandering Group Cambridge, MA
VRDI Instructor *Summer 2018, 2019*

- Organized and led student research groups during an eight week summer program on political redistricting for 80+ graduate and undergraduate students. Met with students daily and both generated and supervised a wide variety of research projects in computational, mathematical, and political topics.

Tufts University Medford, MA
Co-Instructor *Spring 2019*

- Co–taught STS 10: Reading Lab on Mathematical Models in Social Context. This is a reading and discussion based course focused on providing an STS perspective to students who are taking technically-focused modeling classes.

Massachusetts Institute of Technology Cambridge, MA
IAP Instructor *January 2019*

- Developed a four-week course on computational methods for political redistricting. The course incorporated cutting edge mathematical and computational techniques for analyzing gerrymandering.

Dartmouth College Hanover, NH
Instructor *September 2015 - May 2018*

- Designed syllabi and daily lectures. Wrote and graded homework, quizzes, and exams. Fully responsible for course content and material.

Math 36/QSS 36 - Mathematical Modeling in the Social Sciences *Fall 2017*

Data driven course exploring mathematical models and analysis techniques

UNSG 100 - Graduate Ethics Seminar *Fall 2017, 2016, 2015*

Seminar on ethical and professional issues in science and mathematics

Math 8 - Calculus of Functions of one and Several Variables *Winter 2017*

Second term calculus course covering infinite series, vector functions, and partial derivatives

Math 1 - Calculus with Algebra *Fall 2015*

Introductory calculus course with an emphasis on limits and differentiation

Teaching Assistant *September 2013 - June 2015*

· Held tutorial sessions three times per week. Graded quizzes and exams. Designed computing assignments and tutorials for linear algebra.

Math 23 - Differential Equations *Spring 2015*
Math 22 - Linear Algebra with Applications *Fall 2014*
Math 3 - Calculus *Winter 2014*
Math 12 - Calculus Plus *Fall 2013*

Washington State University Pullman, WA
Undergraduate Teaching Assistant *August 2012 - May 2013*

· Held tutorial sessions and graded homework and exams. Supervised a mathematical computing lab.

Math 320 - Modern Algebra *Spring 2013*
Math 330 - Secondary Teaching *Spring 2013*
Math 315 - Differential Equations *Fall 2012*

EDUCATIONAL OUTREACH

UW Data Science for Social Good Seattle, WA
Project Lead *Summer 2021*

· Designed and supervised a research project for four data science fellows on applications of ensemble methods to initial districting plan evaluation. The fellows gave a public presentation of their work and developed a user guide "Applying GerryChain: A Users Guide for Redistricting Problems" with accompanying website, case studies, and code examples to demonstrate good modeling practives and support other researchers working on these problems.

New Hampshire State Math Team Manchester, NH
Math Team Coach *Fall 2018–2020*

· Designed practice problems and preparatory exercises for the AMC exams, ARML, MMATH, and HMMT. Led monthly problem solving sessions and group activities.

LaTeX Workshops Hanover, NH
Organizer *Fall 2016–May 2018*

· Designed and presented a series of eleven one hour–long and two three hour–long workshops on mathematical typesetting in LaTeX with D. Freund and K. Harding. Resources and lesson plans

Crossroads Academy Math Team Lyme, NH
Math Team Coach *September 2015 – May 2018*

· Designed practice problems and preparatory exercises for the AMC exams, MathCounts, and MathLeague. Led weekly problem solving sessions and group activities. During 2015–17, the Crossroads team twice won the Chapter and State MathCounts and MathLeague competitions and placed first in Northern New England on the AMC-8.

New Hampshire State MathCounts Team Lyme, NH
Math Team Coach *March 2017 – May 2017*

· Designed practice problems and preparatory exercises for the national MathCounts exam. Led biweekly problem solving sessions and group activities. Students competed in the national competition in Orlando, Florida.

Johns Hopkins Center for Talented Youth Science and Technology Series Hanover, NH
Workshop Leader

· Developed and presented hour–long workshops for high school students.

Modern Cryptography (with D. Freund) *October 2014*
Forensic Accounting *April 2016*
Binary and Barcodes (with D. Freund) *April 2017*

Dartmouth College Exploring Mathematics Camp Hanover, NH
Co-Instructor

· Organized and presented week long math camps for high school students.
Mathematics of Games *August 2015*
Cryptography *July 2015*

RESEARCH PRESENTATIONS

Talks

1. Analysis Seminar, Pullman, WA *December 2021*
   Introduction to Graphons I and II
2. PPPA Research Colloquium, Pullman, WA *November 2021*
   Computational Methods for Evaluating Districting Plans
3. INFORMS Annual Meeting, Zoom *October 2021*
   Algorithms And Analysis For Centered Redistricting Plans
4. WSU Math Club, Pullman, WA *October 2021*
   Graphs, Geometry, and Gerrymandering
5. Civic Hackathon, Madison, WI *September 2021*
   Introduction to Computational Redistricting
6. Harvard Redistricting Algorithms, Law, and Policy Cambridge, MA *September 2021*
   Technical State of the Art for Computational Redistricting
7. ASA Joint Statistical Meeting, Zoom *August 2021*
   Computational Methods for Assessing Political Redistricting Reforms
8. New Mexico Redistricting Commission, Santa Fe, NM *July 2021*
   Markov chain ensemble metrics for evaluation of redistricting plans
9. Colorado College Summer Program, Colorado Springs, CO *June 2021*
   Computational Redistricting Analysis
10. WSU Seminar in Statistics, Pullman, WA *April 2021*
    Ensemble Analysis for the 2020 Redistricting Cycle
11. Princeton Gerrymandering Project, Princeton, NJ *March 2021*
    Computational Redistricting in 2021
12. Combinatorics, Linear Algebra, and Number Theory, WSU, Pullman, WA *March 2021*
    Gerry-Matchings and Pair-y-Mandering
13. JMM 2021, Washington DC *January 2021*
    Short Course: Mathematical and Computational Methods for Complex Social Systems
14. INFORMS Special Session on Fairness in Operations Research, Baltimore, MD *November 2020*
    Computational Methods For Assessing Districting Plans
15. WSU Seminar in Statistics, Pullman, WA *November 2020*
    Statistical and Computational Methods for Assessing Political Redistricting
16. Pi MU Epsilon Lecture, St. Michael's College, Colchester, VT *October 2020*
    Graphs, Geometry, and Gerrymandering
17. ADSA Annual Meeting, Zoom *October 2020*
    Geospatial Data for Political Redistricting Analysis

18. Common Experience Lecture, Texas State University, San Marcos, TX *October 2020*
Graphs, Geometry, and Gerrymandering
19. Combinatorics, Linear Algebra, and Number Theory, WSU, Pullman, WA *September 2020*
Representations of $SL_2(\mathbb{Z}/p^n\mathbb{Z})$ and spectral properties of Bethe trees
20. CGAD-GTOpt Seminar, Washington State University, Pullman, WA, *July 2020*
Geometric and Optimization Problems Motivated by Political Redistricting
21. Redistricting Conference 2020, Duke University, Durham, NC, *March 2020*
Multiresolution Redistricting Algorithms
22. Math Department Colloquium, College of Charleston, Charleston, SC. *February 2020*
Geospatial Data, Markov Chains, and Political Redistricting
23. Math Department Colloquium, Washington State University, Pullman, WA. *January 2020*
Geospatial Data, Markov Chains, and Political Redistricting
24. JMM 2020, Denver, CO. *January 2020*
Markov chains for sampling connected graph partitions
25. Math Department Colloquium, Pacific University, Forest Grove, OR. *January 2020*
The Mathematics of Nested Legislative Districts
26. MIT Graphics Annual Retreat, North Falmouth, MA. *October 2019*
Connected Graph Partitions and Political Districting
27. Topology, Geometry and Data Seminar, Ohio State University, Columbus, OH. *September 2019*
Hardness results for sampling connected graph partitions with applications to redistricting
28. Math Department Colloquium, Denison University, Granville, OH. *September 2019*
Graphs, Geometry, and Gerrymandering
29. Math Department Colloquium, Oberlin College, Oberlin, OH. *September 2019*
Graphs, Geometry, and Gerrymandering
30. Math Department Colloquium, College of Wooster, Wooster, OH. *September 2019*
Graphs, Geometry, and Gerrymandering
31. Math Monday Colloquium, Kenyon College, Gambier, OH. *September 2019*
Graphs, Geometry, and Gerrymandering
32. Applied Math Seminar, University of Massachusetts Lowell, Lowell, MA. *September 2019*
Hardness results for sampling connected graph partitions with applications to redistricting
33. Math Department Colloquium, Yale University, New Haven, CT. *August 2019*
Mathematical Challenges in Neutral Redistricting
34. Voting Rights Data Institute Seminar, Cambridge, MA. *June 2019*
A Friendly Introduction to Discrete MCMC
35. Voting Rights Data Institute Seminar, Cambridge, MA. *June 2019*
Graphs and Networks: Discrete Approaches to Redistricting
36. Math Department Colloquium, Dartmouth College, Hanover, NH. *April 2019*
Total Variation Isoperimetric Profiles and Political Redistricting
37. ACM Seminar, Dartmouth College, Hanover, NH. *April 2019*
Hardness results for sampling connected graph partitions with applications to redistricting
38. Unrig Summit Masterclass, Nashville, TN. *March 2019*
Legal and Math Deep Dive: Gerrymandering and Redistricting
39. MIT Graphics Seminar, Cambridge, MA. *March 2019*
Computational Challenges in Neutral Redistricting
40. JMM 2019, Baltimore, MD. *January 2019*
Matched Products and Stirling Numbers of Graphs
41. Societal Concerns in Algorithsm and Data Analysis, Weizmann Institute of Science, Rehovot, Israel. *December 2018*
Computational Problems in Neutral Redistricting
42. Math and Law of Redistricting, Radcliffe Institute, Cambridge, MA. *December 2018*
GerryChain and MCMC tutorials
43. Math Colloquium, Tufts University, Medford, MA. *November 2018*
Matched Products and Stirling Numbers of Graphs

44. MIT Graphics Annual Retreat, Dedham, MA. *October 2018*
Mathematical Challenges in Neutral Redistricting
45. SAMSI Workshop on Quantitative Redistricting, Duke University, Durham, NC. *October 2018*
Compactness Profiles and Reversible Sampling Methods for Plane and Graph Partitions
46. Election Teach–in, SMFA, Boston, MA. *October 2018*
Computational Challenges in Political Redistricting
47. STS Seminar, Tufts University, Cambridge, MA. *September 2018*
Mathematical Modeling of Social Connections
48. Voting Rights Data Institute Seminar, Cambridge, MA. *June 2018*
Introduction to Monte Carlo Methods
49. Mathematics Colloquium, University of Central Florida, Orlando, FL. *February 2018*
Dynamical Models for Multiplex Data
50. Mathematics Colloquium GVSU, Grand Valley, MI. *February 2018*
Random Walk Null Models for Time Series
51. Omidyar Fellowship Presentation, Santa Fe, NM. *January 2018*
Mathematical Embeddings of Complex Systems
52. Mathematics Colloquium at University of San Fransisco, San Fransisco, CA. *January 2018*
Dynamical Models for Multiplex Data
53. Mathematics Colloquium at Providence College, Providence, RI. *January 2018*
Dynamical Models for Multiplex Data
54. JMM, San Diego, CA. *January 2018*
Dynamical Modeling for Multiplex Networks
55. International Complex Networks Conference Lyon, France. *December 2017*
Multiplex Dynamics on the World Trade Web
56. Physics Colloquium at Washington University, St. Louis, MO. *October 2017*
Spectral Clustering on Multiplex Data
57. SIAM Annual Meeting, Pittsburgh, PA. *July 2017*
Permutation Complexity Measures for Time Series
58. Applied and Computational Mathematics Seminar, Hanover NH. *November 2016*
Random Dot Product Models for Weighted Networks
59. Inference on Networks: Algorithms, Phase Transitions, New Models and New Data, Santa Fe, NM. *December 2015*
Dynamically Motivated Models for Multiplex Networks
60. Applied Math Days, Troy, NY. *April 2015*
Multiplex Structure on the World Trade Web
61. Graduate Student Combinatorics Conference, Lexington, KY. *March 2015*
Total Dynamics on Multiplex Networks
62. Sixteenth International Fibonacci Conference, Rochester, NY. *July 2014*
Enumerating Distinct Chessboard Tilings
63. Dartmouth Graduate Student Seminar, Hanover, NH. *(Quarterly) 2013 - 2018*
Various Topics
64. Joint Mathematics Meeting, San Diego, CA. *January 2013*
Counting Combinatorial Rearrangements, Tilings with Squares and Symmetric Tilings
65. West Coast Number Theory Conference, Asilomar, CA. *December 2012*
Generalized Lucas Bases
66. Young Mathematician's Conference, Columbus, OH. *July 2012*
Combinatorial Rearrangements on Arbitrary Graphs
67. Northwest Undergraduate Mathematics Symposium, Portland, OR. *March 2012*
Combinatorial Rearrangements on Arbitrary Graphs
68. WSU Graduate Seminar on Combinatorial Geometry, Pullman, WA. *(Quarterly) 2012-2013*
Various Topics

Posters

1. SIAM Workshop on Network Science, Boston, MA. *July 2016*
   Generalized Random Dot Product Models For Multigraphs
2. Dartmouth Graduate Student Poster Session, Hanover, NH. *April 2016*
   Generalized Dot Product Models for Weighted Networks
3. Dartmouth Graduate Student Poster Session, Hanover, NH. *April 2015*
   Multiplex Structures in the World Trade Web
4. WSU SURCA, Pullman, WA. *March 2013*
   Empirical Analysis of Space Filling Curves for Scientific Computing Applications
5. WSU SURCA, Pullman, WA. *April 2012*
   Combinatorial Rearrangements, Restricted Permutations, and Matrix Permanents

HONORS AND AWARDS

- Dartmouth Hannah Croasdale Award *2018*
  College-wide award for the graduating Ph.D. student that best exemplifies the qualities of a scholar.
- Dartmouth Graduate Student Teaching Award *2017*
  College-wide award for the graduate student who best exemplifies the qualities of a college educator.
- Dartmouth Graduate Fellowship *2014–18*
- NSF Graduate Research Fellowship: Honorable Mention *2014, 2015*
- Dartmouth GAANN Fellowship *2013*
- WSU Morris Knebelman Outstanding Senior Award *2013*
- WSU Department of Mathematics Outstanding Senior *2013*
- WSU Emeritus Society Award in the Physical Sciences *2013*
- WSU J. Russell and Mildred H. Vatnsdal Memorial Scholarship *2013*
- WSU SURCA Crimson Award: Computer Science and Mathematics *2012, 2013*
- WSU Auvil Undergraduate Scholars Fellowship *2012*
- WSU Leonard B. Kirschner Scholarship *2012*
- WSU College of Sciences Undergraduate Research Grant *2012*
- Norma C. Fuentes and Gary M Kirk Award for Excellence in Undergraduate Research *2012*

PROFESSIONAL SERVICE

Peer Reviewer

- Election Law Journal
- Transactions on Signal and Information Processing over Networks
- Multiscale Modeling and Simulation: A SIAM Interdisciplinary Journal
- International Conference on Learning Representations (ICLR)
- International Conference on Artificial Intelligence and Statistics (AISTATS)
- AAAI Conference on Artificial Intelligence (AAAI)
- International Conference on Machine Learning (ICML)
- ACM-SIAM Symposium on Discrete Algorithms (SODA)
- Neural Information Processing Systems (NeurIPS)
- Transactions on Pattern Analysis and Machine Intelligence (TPAMI)
- Chaos: An Interdisciplinary Journal of Nonlinear Science
- Involve: A Journal of Mathematics
- Entropy
- MATCH Communications in Mathematical and in Computer Chemistry

Appendix B

Data and Materials

This appendix describes the data and materials that I relied on while performing this analysis and crafting this report.

B.i Data

The primary data sources and document repositories for the analysis in this report are publicly available, including the underlying geospatial data. I made use of data and documents from the following sources:

- Wisconsin-specific geospatial data and annotations (https://legis.wisconsin.gov/ltsb/gis/data/)
- Geospatial and population data from the US Census (https://www.census.gov/geographies/mapping-files/time-series/geo/tiger-line-file.html)
- Filings in this case (https://www.wicourts.gov/courts/supreme/origact/2021ap1450.htm)
  - Briefs, reports, maps, and expert materials submitted by the parties on December 15, 2021 including material produced by parties pursuant to agreement on discovery
  - Supreme Court's November 30 order
- Election and CVAP data prepared by counsel that I merged with the LTSB Ward data

B.ii Computational Libraries

The bulk of the computational work for this report was carried out using standard libraries of the Python programming language. I also used the following more specialized packages for specific computational tasks.

- [MAUP] github.com/mggg/maup
- [Gerrychain] github.com/mggg/gerrychain
- [Geocompactness] github.com/leehach/geocompactness