IN THE SUPREME COURT OF WISCONSIN

No. 2021AP001450 OA

BILLIE JOHNSON, ERIC O'KEEFE, ED PERKINS and RONALD ZAHN,

Petitioners,

BLACK LEADERS ORGANIZING FOR COMMUNITIES, VOCES DE LA FRONTERA, LEAGUE OF WOMEN VOTERS OF WISCONSIN, CINDY FALLONA, LAUREN STEPHENSON, REBECCA ALWIN, CONGRESSMAN GLENN GROTHMAN, CONGRESSMAN MIKE GALLAGHER, CONGRESSMAN BRYAN STEIL, CONGRESSMAN TOM TIFFANY, CONGRESSMAN SCOTT FITZGERALD, LISA HUNTER, JACOB ZABEL, JENNIFER OH, JOHN PERSA, GERALDINE SCHERTZ, KATHLEEN QUALHEIM, GARY KRENZ, SARAH J. HAMILTON, STEPHEN JOSEPH WRIGHT, JEAN-LUC THIFFEAULT, and SOMESH JHA,

Intervenors-Petitioners,

v.

WISCONSIN ELECTIONS COMMISSION, MARGE BOSTELMANN in her official capacity as a member of the Wisconsin Elections Commission, JULIE GLANCEY in her official capacity as a member of the Wisconsin Elections Commission, ANN JACOBS in her official capacity as a member of the Wisconsin Elections Commission, DEAN KNUDSON in his official capacity as a member of the Wisconsin Elections Commission, ROBERT SPINDELL, JR. in his official capacity as a member of the Wisconsin Elections Commission and MARK THOMSEN in his official capacity as a member of the Wisconsin Elections Commission,

Respondents,

THE WISCONSIN LEGISLATURE, GOVERNOR TONY EVERS, in his official capacity, and JANET BEWLEY SENATE DEMOCRATIC MINORITY LEADER, on behalf of the Senate Democratic Caucus,

Intervenors-Respondents.

REBUTTAL EXPERT REPORT OF DR. DARYL DEFORD ON BEHALF OF INTERVENORS-PETITIONERS CITIZEN MATHEMATICIANS AND SCIENTISTS

Rebuttal Expert Report of Daryl R. DeFord on behalf of the Citizen Mathematicians and Scientists

January 4, 2022

I Executive Summary

On December 30, 2021, I submitted a responsive report in this proceeding analyzing congressional and state legislative redistricting proposals ("First Report"). After submitting that report, counsel for the Citizen Mathematicians and Scientists ("CMS") asked me to review and analyze any responsive briefs and expert reports submitted by other parties in this proceeding. During this process, I examined specific claims about the performance of the CMS maps on constitutional, statutory, and traditional redistricting criteria. I summarize my conclusions as follows:

- **State Legislative Maps.** In my First Report, I evaluated alternatives to the CMS state legislative plans and concluded that all fell short of the CMS plans, which makes necessary modifications to district boundaries to achieve lower population deviation, fewer county splits, and extremely competitive compactness scores, among other things. After reviewing the responsive materials filed on December 30, 2021, nothing causes me to alter that conclusion. Although several parties disregard the CMS plans because of their performance on a single least change metric, see, e.g., Legislature's Response Br.7; BLOC Response Br. 23, I find that the CMS plans best navigate complex tradeoffs created by Wisconsin constitutional requirements, and that other parties fail to appropriately weight performance of the CMS proposal on the requirements.

  To begin with, some of the responses submitted by the parties elide the importance of population deviation. Figures 1 and 2 below illustrate that the CMS state legislative maps perform best on this important metric, in that they come closest to approximating exact population balance. *See* Nov. 30, 2021 Order ("Order") at 28.

Figure 1: Population Deviation in Proposed Assembly Plans

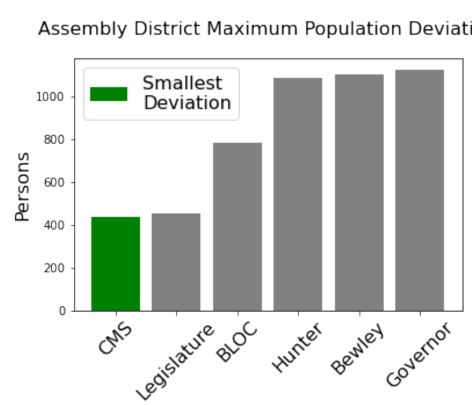
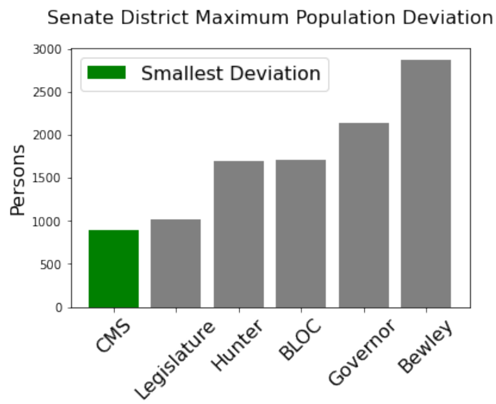
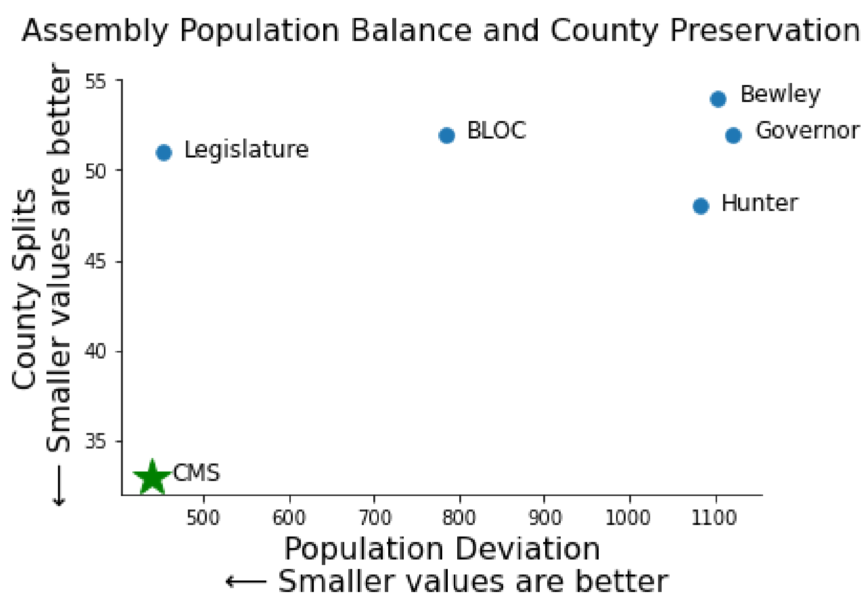


Figure 2: Population Deviation in Proposed Senate Plans



Many responsive briefs and expert reports submitted by the parties also fail to recognize the tension between efforts to reduce population deviation and reduce splits. Figures 9 and 10[1] below illustrate that the CMS plans navigate that tension more effectively than other parties, achieving the lowest range of deviation between their most over- and under-populated district, and also creating the smallest number of county splits.[2]

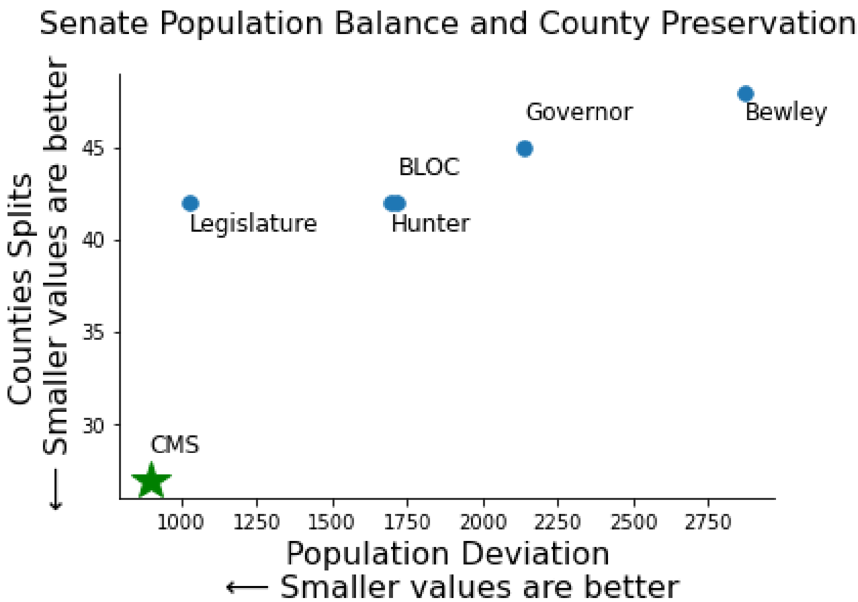
Figure 9: Comparison of Population Balance & County Splits in Assembly Maps



[1]Figures in the Executive Summary are numbered as they appear below in Section IV.

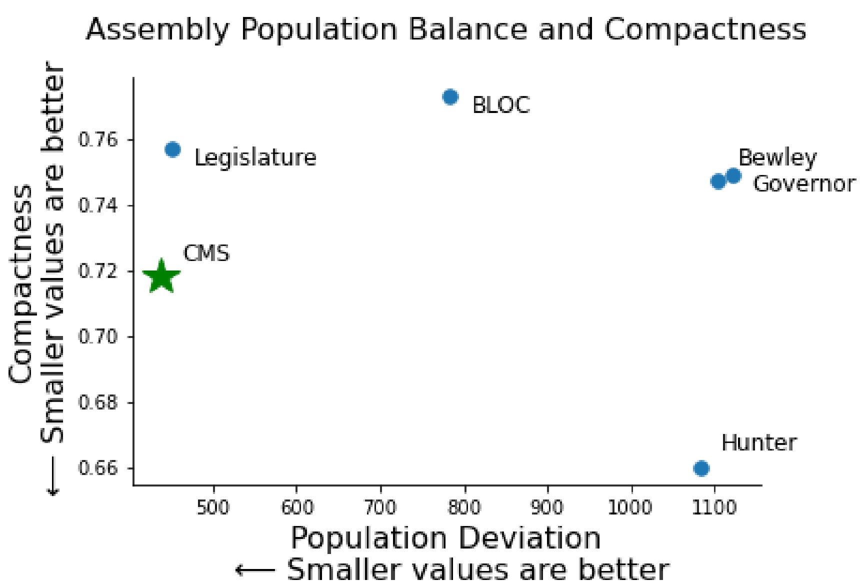
[2]For precision, these figures report the number of county splits in excess of those required because county population surpasses the ideal district size. However, even if all county splits are included, the CMS plans achieve the fewest splits.

Figure 10: Comparison of Population Balance & County Splits in Senate Maps



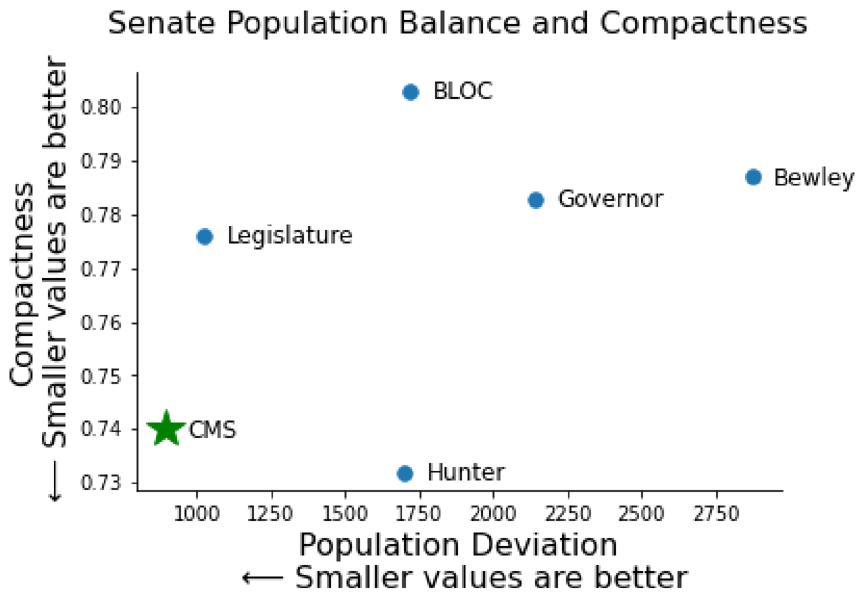
Many responsive briefs and expert reports submitted by the parties also fail to recognize the tension between efforts to reduce population deviation or splits, one the one hand, and compactness, on the other. *But see* Rebuttal Report of Dr. Kenneth R. Mayer at 1. Notably, the CMS state legislative plans navigate this challenge very effectively. As Figures 11 and 12 illustrate, the CMS state legislative plans manage to perform best on population equality while achieving the second highest mean Polsby-Popper score.[3]

Figure 11: Comparison of Population Balance & Compactness in Assembly Maps



[3]To normalize this plot, I calculate the complement of the mean Polsby-Popper score by subtracting that score from one. This means that, contrary to ordinary practice, lower scores are better.

Figure 12: Comparison of Population Balance & Compactness in Senate Maps



Based on my analysis of these metrics, and my understanding of applicable law, I conclude that parties are wrong to disregard the CMS plans just because of their performance on a single least change metric. Because the CMS plans successfully navigate the tradeoffs between redistricting criteria that are required by the Wisconsin Constitution, I find that they merit selection.

- **Congressional Maps.** Nothing in the responsive briefs and reports submitted on congressional plans alters my initial conclusion: That alternatives to the CMS plan do not perform as effectively under the applicable framework. In fact, if respect for municipal boundaries (and, by extension, county boundaries) is "probative" of consistency with Wisconsin political geography and least change, Congressmen's Responsive Br. 20, then that only strengthens my conclusion that the CMS plan performs very well across the relevant criteria. I say that because the CMS plan performs extremely well on preservation of county and ward lines, including because it preserves several whole counties that are split in the 2011 enacted map and remain split in other proposed maps, as demonstrated in Section V.A.

II Qualifications and Assignment

My qualifications are set out in my previous expert report, filed on December 30, 2021. A full copy of my CV, containing a list of my publications in the last 10 years, is reattached as Appendix A, for ease of review. I continue to be compensated at a rate of $300 per hour and, as indicated in my First Report, my compensation does not depend in any way on the results of my analysis, the conclusions that I draw, or the eventual outcome of the litigation. I have not testified as an expert at deposition or trial in the last four years.

III Assignment

After the submission of my First Report, counsel for CMS asked that I review and analyze the briefs and expert reports submitted on December 30, 2021. With respect to congressional plans, I reviewed and analyzed briefs and reports submitted by the Congressmen, the Governor, the Hunter Plaintiffs, and the Johnson Plaintiffs. With respect to the state legislative plans, I reviewed and analyzed briefs and reports submitted by the Legislature, the Governor, the BLOC Plaintiffs, the Hunter Plaintiffs, and the Johnson Plaintiffs.[4]

In reviewing these materials, I continued to apply the quantitative measures associated to the districting criteria discussed in Part IV of my First Report. The conclusions set out here are based on my analysis of the materials submitted and produced by the parties on December 30, 2021, as well as data and materials originally compiled for my first report. A complete list of the materials relied on in forming the opinions stated in this rebuttal report is attached as Appendix B.

IV Analysis of State Legislative Proposals

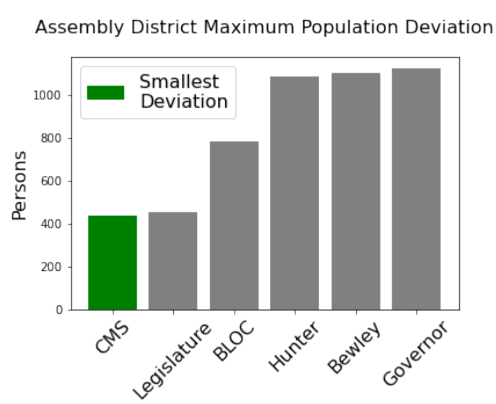
After reviewing the December 30, 2021 briefs and reports, I stand by the conclusions and measurements presented in my First Report. In this Section, I address claims concerning the CMS senate and assembly proposals, as well as omissions in the responses submitted on December 30, 2021, focusing specifically on the criteria from the Wisconsin State Constitution.

IV.A Population Deviation

Solving the problem of malapportionment in the current districts and minimizing the population deviation in the proposed maps is an important constitutional criterion. Order ¶ ¶ 9-11, 13, 28. However, some parties do not appear to weight differences between performance on this metric, *see* BLOC Response Br. 47; Governor's Response Br. 13; Hunter Response Br. 14, notwithstanding what I understand to be a requirement that parties attempt to approximate exactness on this criterion (as closely as they can). *see* Order 28. As illustrated in Figures 1 and 2 below, the CMS state legislative plans achieve lower top-to-bottom deviation than any alternative plans. Based on my understanding of the law applicable to Wisconsin state legislative redistricting, See Order at 28, this provides a powerful reason to select the CMS plans.

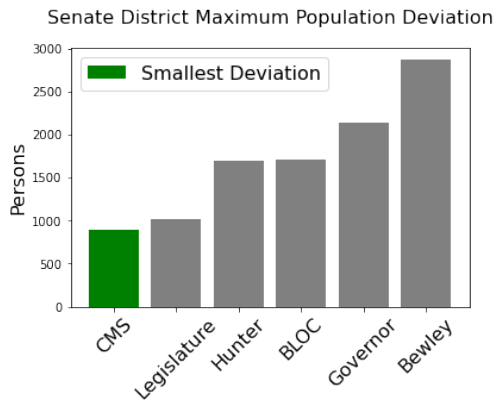
Figures 1 and 2 demonstrate the top-to-bottom deviations of each of the proposed Assembly and Senate plans, sorted from smallest to largest deviation, with the smaller deviation being the better score. In both cases, as detailed in my First Report, Tables 6 and 12, the CMS maps offer the best population balance, followed by the Legislature's plans. All other maps are less well apportioned than these two plans.

Figure 1: Population Deviation in Proposed Assembly Plans



[4]Although I am aware that the Congressmen have submitted a modified map, I was not asked to and did not analyze it in connection with the preparation of this report.

Figure 2: Population Deviation in Proposed Senate Plans



IV.B Preserving City, Town, and County Boundaries

Some parties disregard the performance of CMS plans on the Wisconsin Constitutional requirement that legislative districts respect county, town, and ward lines. *see* Legislature Response Br. 7; BLOC Response Br. 23; Governor's Response Br. 20. As my First Report demonstrated, the CMS plans perform extremely well on this metric. *see* First Report Tables 8 and 14. Additionally, in any population-balanced plan, there are some counties that must be split into several state legislative districts, because their population exceeds the size of an ideal district. According to the 2020 census populations, 25 counties in Wisconsin contain a larger population that an ideal Assembly district and 6 counties contain a larger population than the ideal Senate district. However, as demonstrated in Figures 3 and 4, controlling for county size does not alter the conclusion that CMS plans perform extremely well on this metric. Those figures report the percentage of counties that are split or subdivided only to the degree that their population requires. In each figure, an "Intact" county is one that is not divided any more than is necessary due to its population. As the plots show, the CMS districts significantly outperform all other proposals on this metric, for both Assembly and Senate districts. Specifically, the CMS plan is the only one to preserve over half of the counties in both plans.

Figure 3: Percentage of Intact Counties in Assembly Proposals

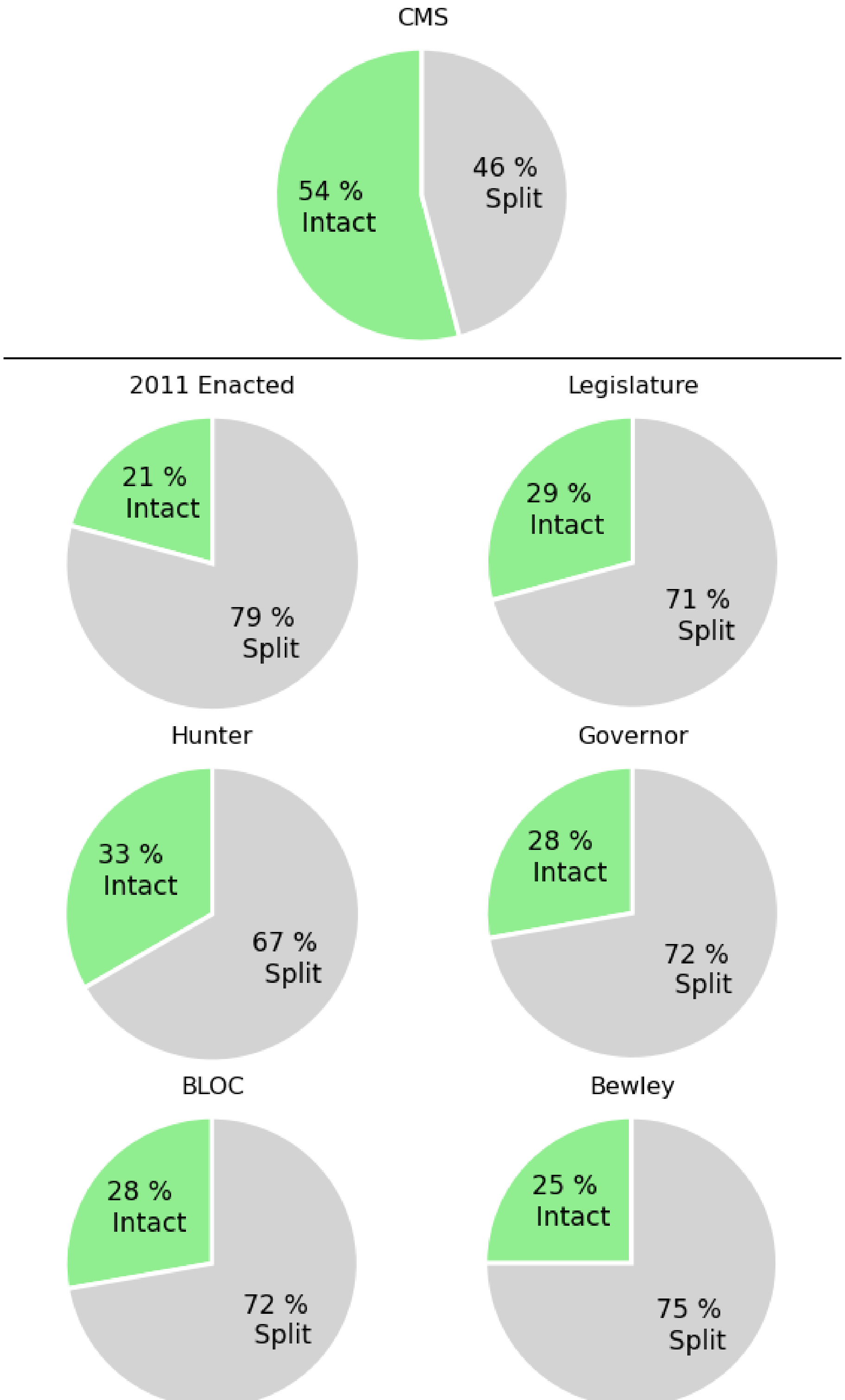
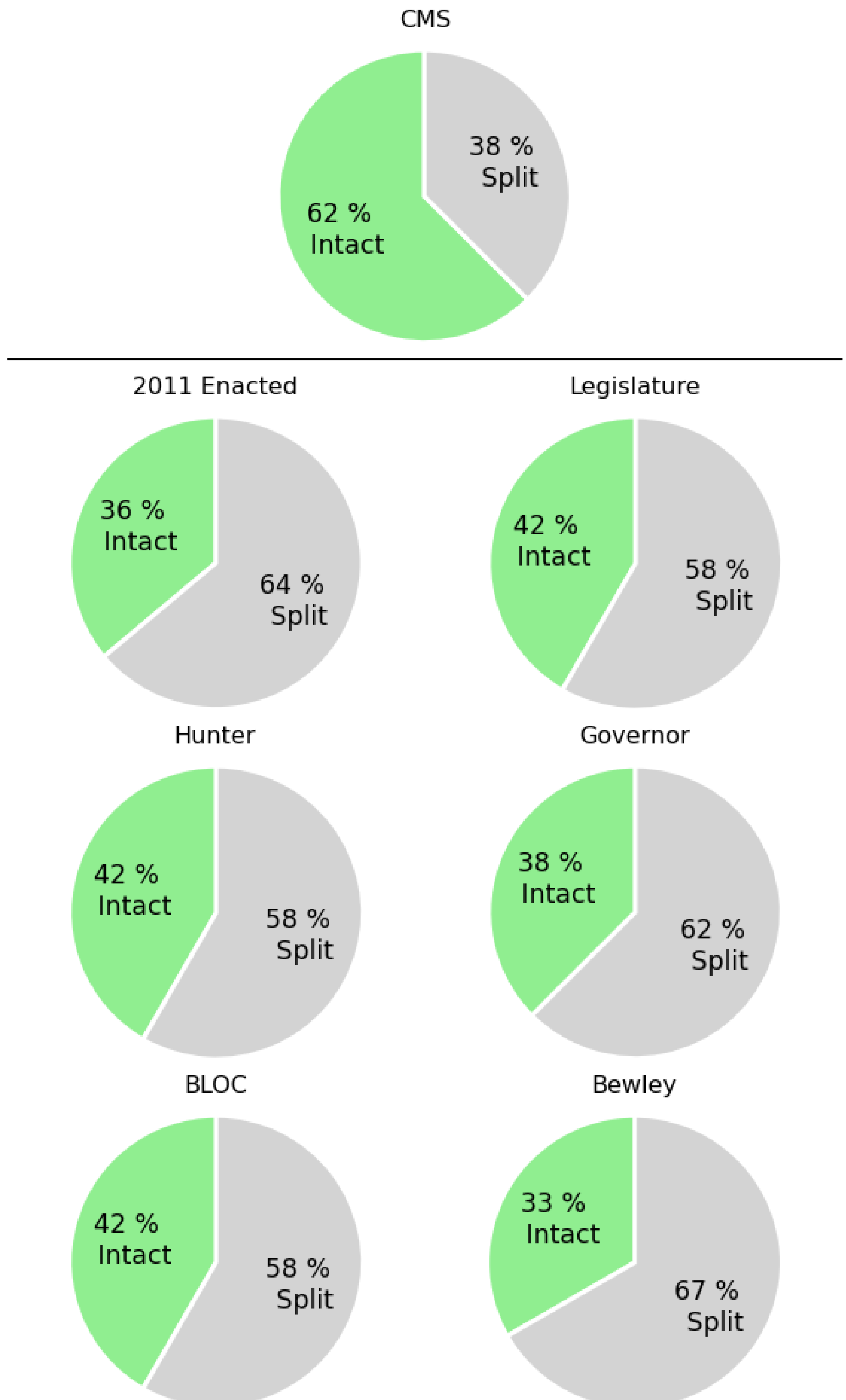


Figure 4: Percentage of Intact Counties in Senate Proposals



To further illustrate what the CMS plans achieve, and why the responsive materials submitted in this proceeding do not alter my conclusion that the CMS plans are particularly strong on this metric, consider the example of Dodge County. With a population of 89,396, Dodge County is necessarily split at least once in every assembly map, but does not need to be split in a Senate map. As illustrated by Figures 5 and 6 below, the CMS plan only splits Dodge County in the Assembly map. There, it divides Dodge County once, creating two pieces, as needed to comply with population deviation requirements. All the alternative assembly plans go farther, splitting Dodge County up to five times, creating six pieces. And in the Senate map, where Dodge County does not need to split at all, some plans nevertheless split the county three times, creating four pieces. Although this is just one example, it is consistent with the broader data, which demonstrate that the CMS plans are superior on metrics associated with the Wisconsin Constitutional requirement to protect county lines. More examples of this type are discussed in Section IV.E below.

Figure 5: Assembly Splits of Dodge County

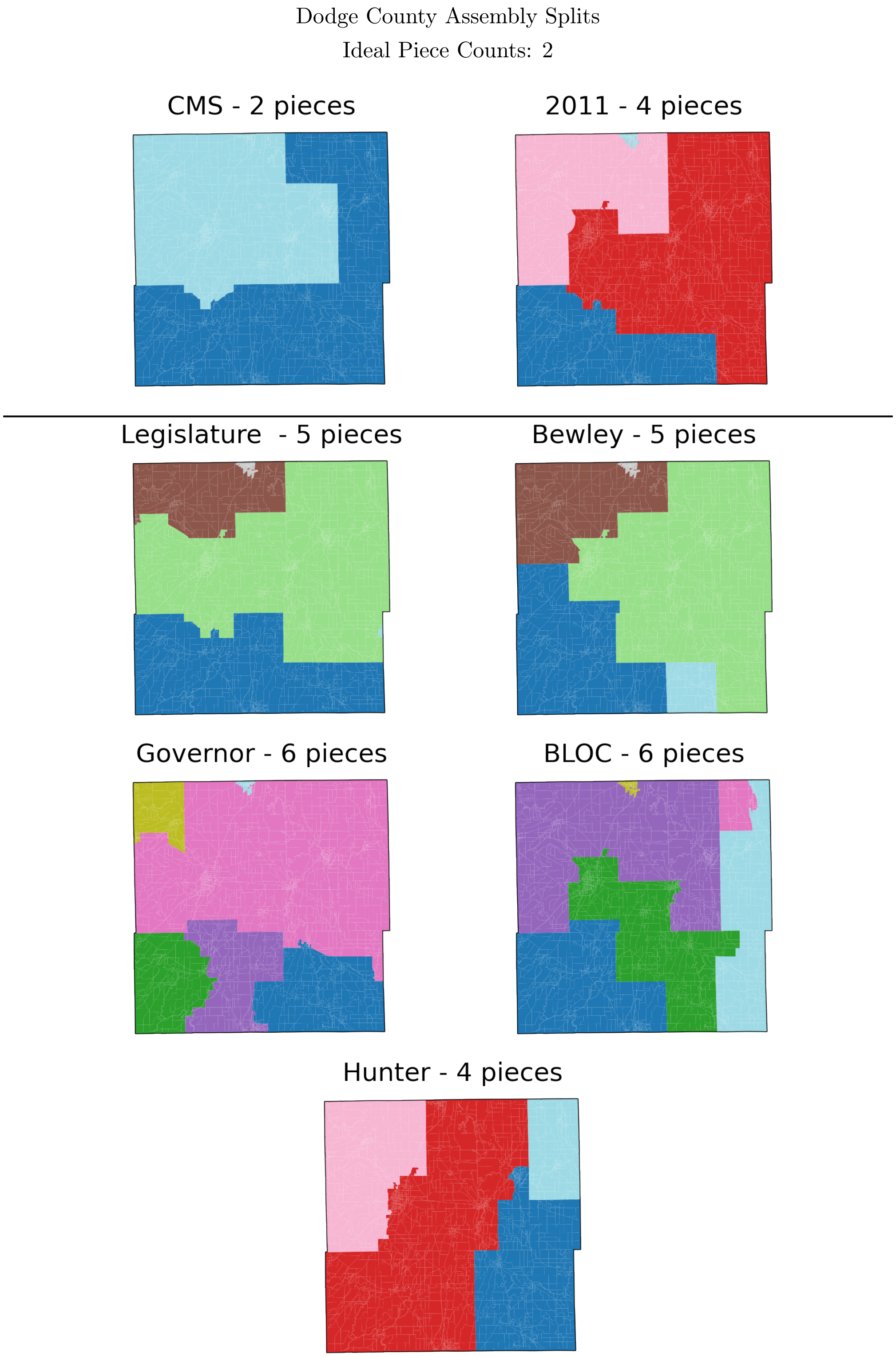
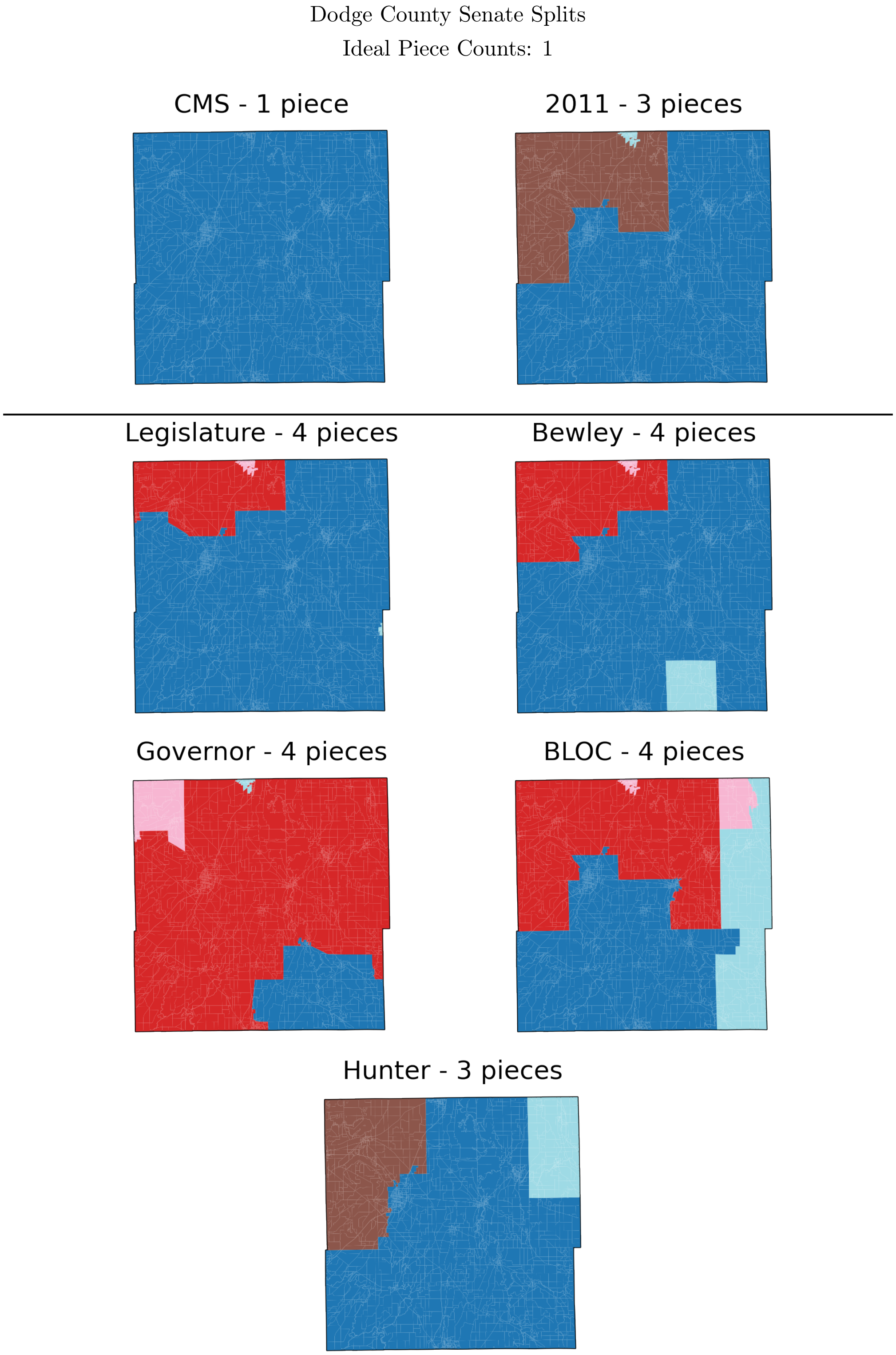


Figure 6: Senate Splits of Dodge County

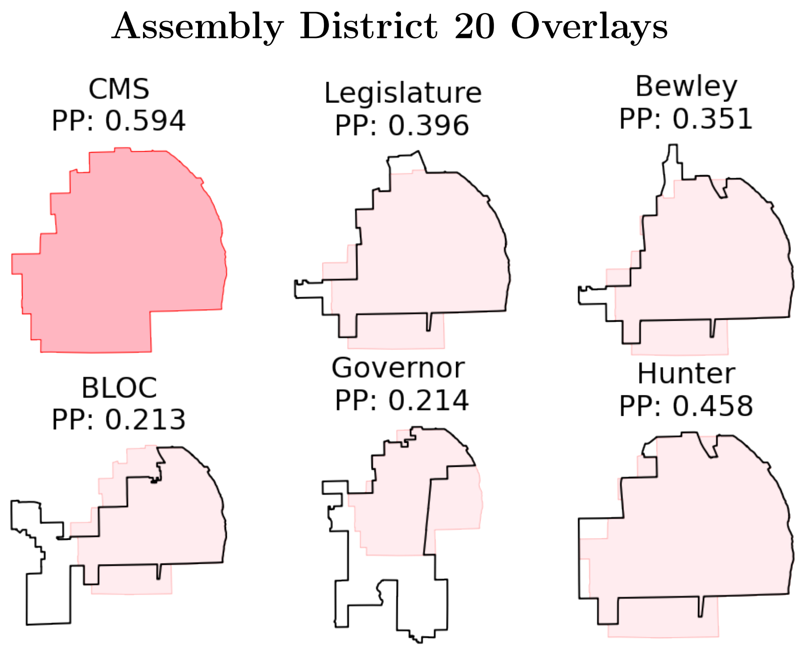


IV.C Compactness

Several parties also disregard the performance of CMS plans on the Wisconsin Constitutional requirement that legislative districts be "as compact as practicable." *See* Legislature Response Br. 7; Governor's Response Br. 19; BLOC Response Br. 23. As demonstrated in my First Report, the CMS Assembly plan performs second best on all of the computed mean continuous metrics of compactness, as well as cut edges. The CMS Senate plan performs similarly, except that it performs best on mean Reock scores.

To illustrate what the CMS plans achieve, and why the responsive materials submitted in this proceeding do not alter my conclusion that the CMS plans are particularly strong on this metric, consider the examples of Assembly districts 20 and 29, as well as Senate districts 10 and 13. As illustrated by Figures 7 and 8 below, and consistent with my understanding of Wisconsin Constitutional requirements, these districts are measurably more compact than their correlates in the alternative plans. Each of these plots shows the boundary of each other proposal's overlapping district of the same number overlayed on the CMS district, together with the corresponding Polsby-Popper score. While these are just a few examples, they reflect that the CMS plans perform extremely well on the Wisconsin Constitutional requirement to maximize compactness to the extent practicable.

Figure 7: Compactness Overlays for Assembly Districts



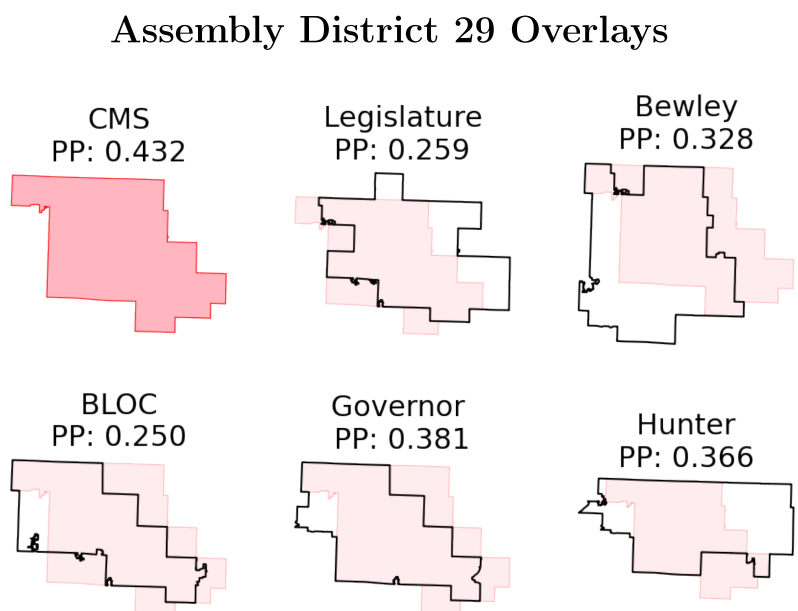
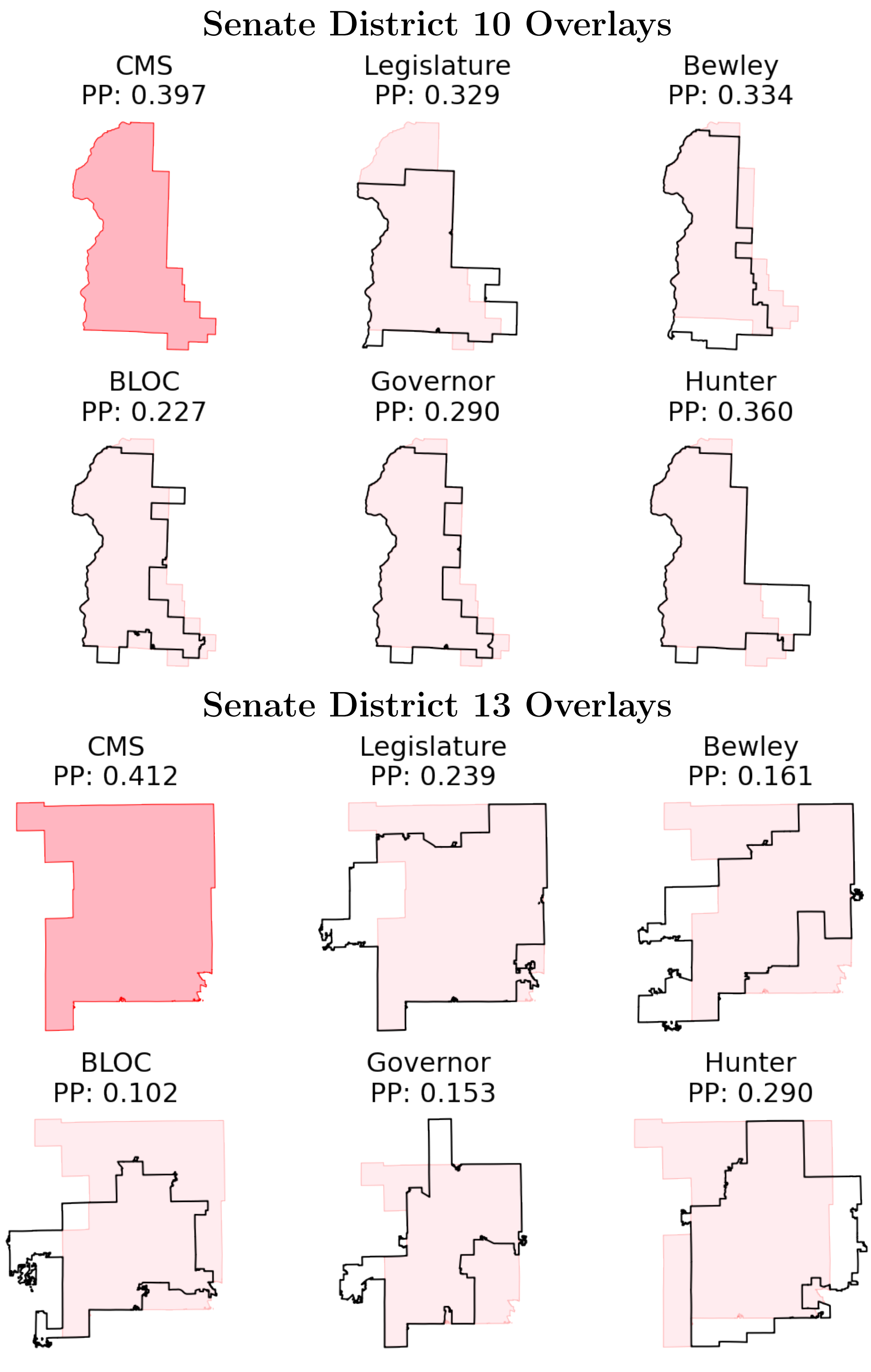


Figure 8: Compactness Overlays for Senate Districts



IV.D The CMS Plans Effectively Manage Tradeoffs Between Wisconsin Constitutional Requirements

Based on my understanding of the law applicable to Wisconsin state legislative redistricting, the decision of some parties to disregard the CMS plans is misplaced. While several parties tout their performance on particular metrics, *see* e.g. Legislature Response Br. 7 (emphasizing strength on population balance); Hunter Response Br. 2 (emphasizing strength on compactness), the CMS plans perform extremely well across all of them. The CMS plans perform best on the Wisconsin constitutional requirement that plans approximate exactness as closely as possible, with respect to population balance. Order ¶ 33. The CMS plans also perform best on the preservation of county lines, splitting fewer counties into fewer pieces on both state legislative maps, without splitting a single ward. First Report 15, 18. And, as discussed above, the CMS plans also perform extremely well on measures of compactness.

It is noteworthy that the CMS plans achieve this level of performance on the required criteria, because modifying districts to improve one criterion can involve tradeoffs on others. Figures 9-12 demonstrate this comparison with respect to county splits and compactness, demonstrating that the CMS plans navigate these tensions more effectively than other proposals, with respect to population deviation on the one hand, and county splits or compactness on the other. Figures 9 and 10 show how the proposed plans perform when accounting for top-to-bottom population deviation and respect for county lines. The top-to-bottom deviation is reported as a gross number of persons, while respect for county lines is reported based on the above those made necessary by the population of the county. The figures confirm that the CMS plans perform best, notwithstanding natural tension between equalizing population and respecting county lines.

Figure 9: Comparison of Population Balance & County Splits in Assembly Maps

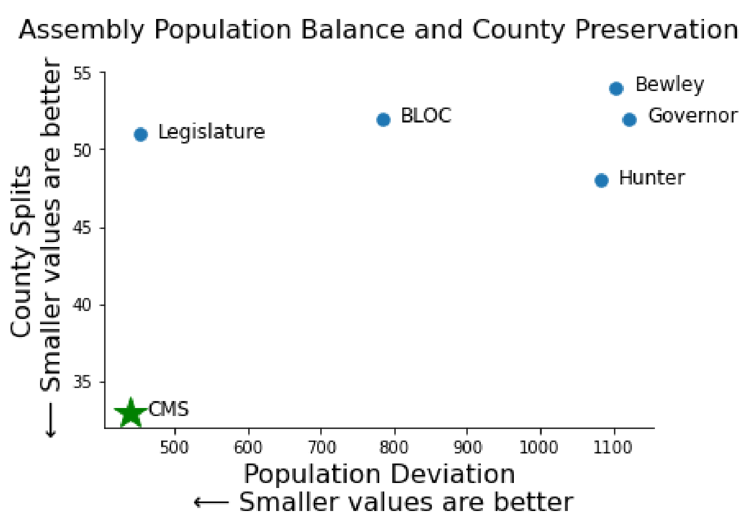
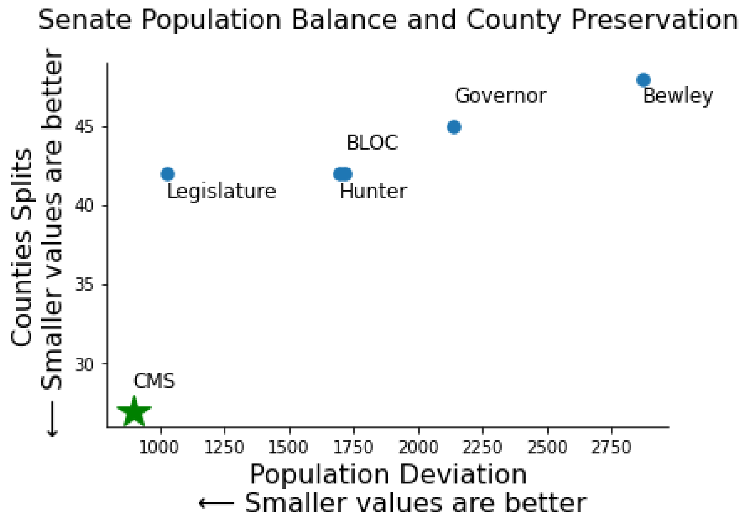


Figure 10: Comparison of Population Balance & County Splits in Senate Maps



Figures 11 and 12 tell a similar story for compactness, demonstrating how the proposed plans perform when accounting for population deviation and the Polsby-Popper measure of compactness. In each plot, we take the complement of the Polsby-Popper metric so that a plan scores better on the compactness measurement if the Polsby-Popper score subtracted from 1 is smaller - that is, plans with better Polsby-Popper values have a smaller value for 1 - Polsby-Popper. The figures demonstrate that the CMS plan does not sacrifice population balance to achieve excellent performance with respect to compactness, and that the only plan to score better on compactness (the Hunter Plan) scores measurably worse on population deviation.

Figure 11: Comparison of Population Balance & Compactness in Assembly Maps

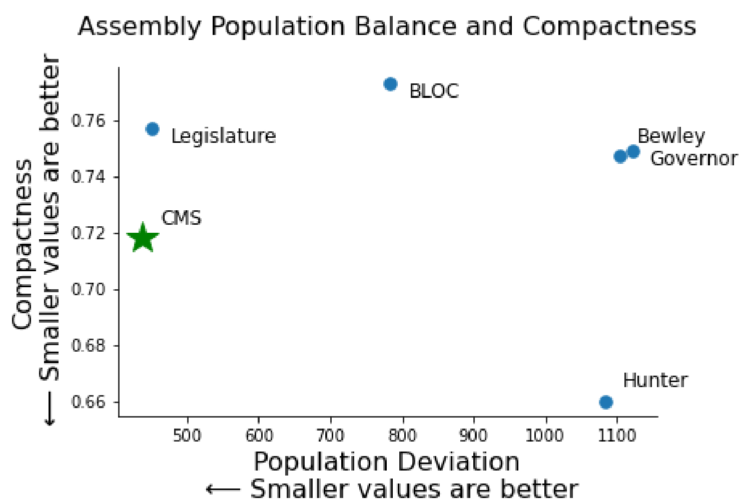
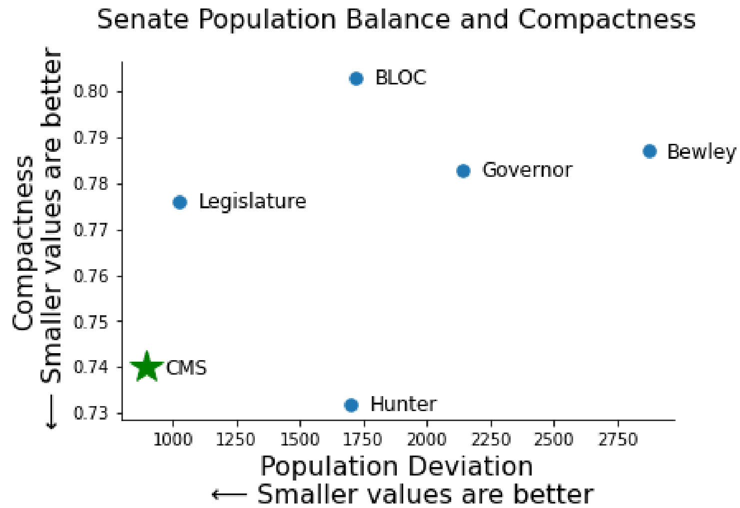
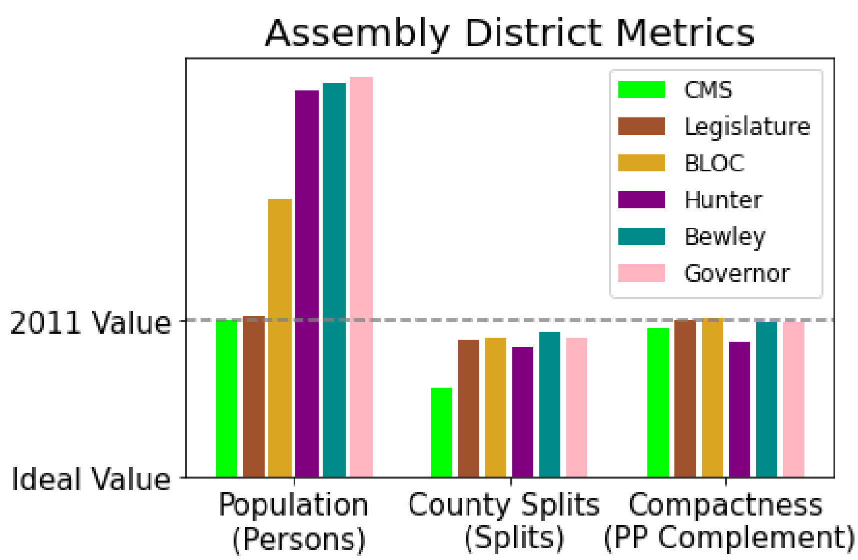


Figure 12: Comparison of Population Balance & Compactness in Senate Maps



It is possible to further distill how the proposed plans perform across all three metrics by normalizing them on a scale between the ideal value for each metric (i.e. zero population deviation, zero county splits, perfect compactness)[5], and values reported for the 2011 enacted plan.[6] As Figures 13 and 14 illustrate, doing so confirms that the CMS plan does the best job balancing these required criteria, as it performs best on deviation and splits, and second best on compactness (trailing a plan that it outperforms significantly on other measures).

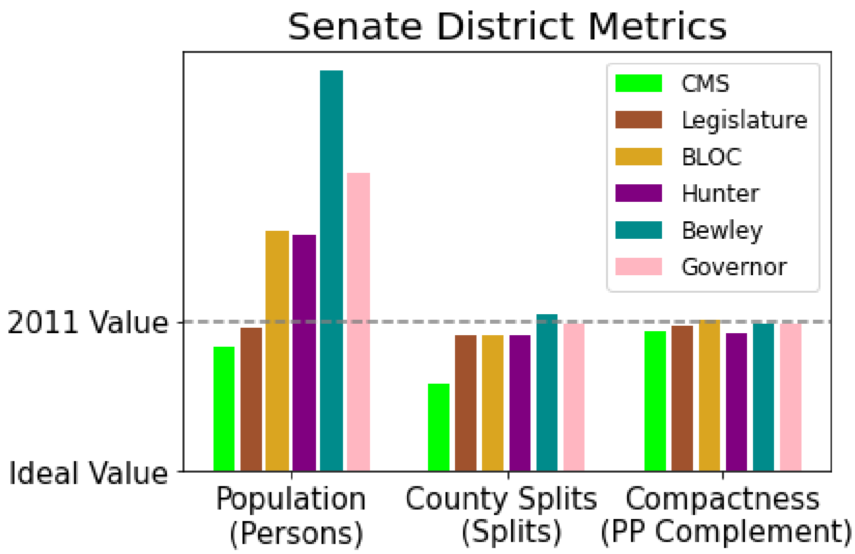
Figure 13: Metric Comparison for Assembly Maps



[5]The ideal values defined below are not necessarily attainable for a plan. For example, it is not possible to construct a plan with an average Polsby-Popper complement score of zero, even splitting census blocks. As with the deviations due to the change in underlying units, this only changes the scaling, not the relative order of plans.

[6]For this purpose, I use figures reported in Appendix B of the Amos report for the Bewley plan.[7] A summary of how I normalized the values and computations reported in this figure is attached as Appendix C to this report.

Figure 14: Metric Comparison for Senate Maps



IV.E Least Change

In the material that I reviewed, several parties disregard the CMS map for its performance on a single least change metric: core retention.[8] Legislature Response Br. 7; Governor Resp. 10. However, based on my review of the proposed plans, that criticism is misguided. As the Legislature recognizes, Legislature Response Br. 6, there are necessary tradeoffs between performance on population deviation (where the CMS plan excels) and core retention. *See* Rebuttal Report of Dr. Kenneth R. Mayer at 1 (observing that "tradeoffs are always necessary among redistricting principles that exist in tension with each other" and citing population equality versus compactness, and compactness versus splits as examples). As discussed above, there are also tradeoffs between protecting county lines and core retention. Figures 15 - 22 illustrate this directly. Each depicts a county that the 2011 enacted map split into more pieces than necessary. As the figures demonstrate, the CMS Assembly and Senate plans reduce or eliminate these splits, which frequently remain in plans proposed by the other parties. The reduction or elimination of those splits necessarily reduces core retention, since it requires moving population from one to another district. These examples also demonstrate instances where a single district from another proposal intersects a county in two discontiguous components.

In Figures 15, 16, 21, and 22 below, the boundary of the county (as contained in the census geography) contains water area not assigned by the plans. This is reflected in the corresponding plots as an area of white territory on the map. These spaces do not represent split pieces of the county and are not counted as such in the computations.

[8]Unless otherwise indicated, I use the term core retention in this report to refer to the movement of people rather than area from one district to another.

Figure 15: Assembly Splits of Fond du Lac County

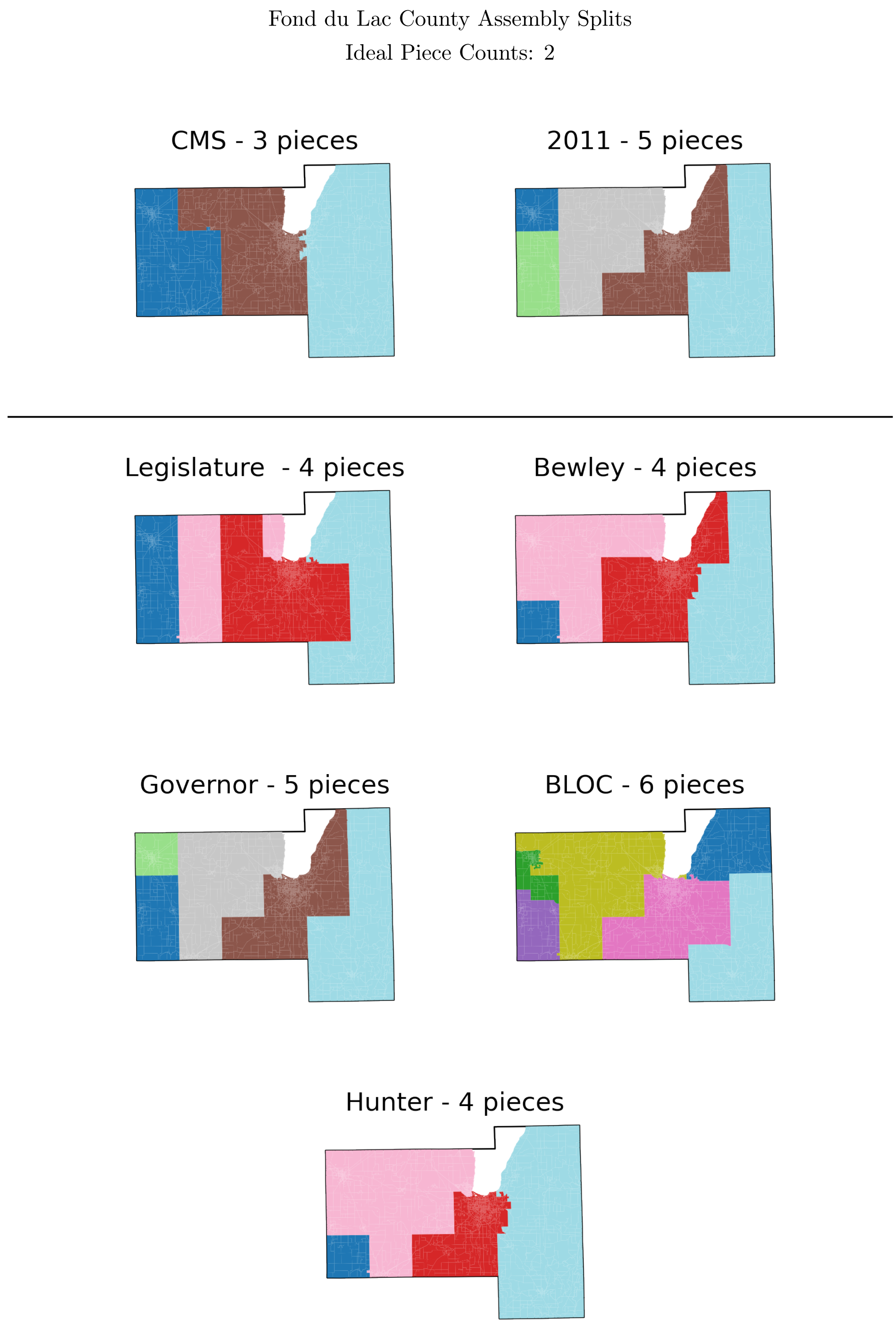


Figure 16: Senate Splits of Fond du Lac County

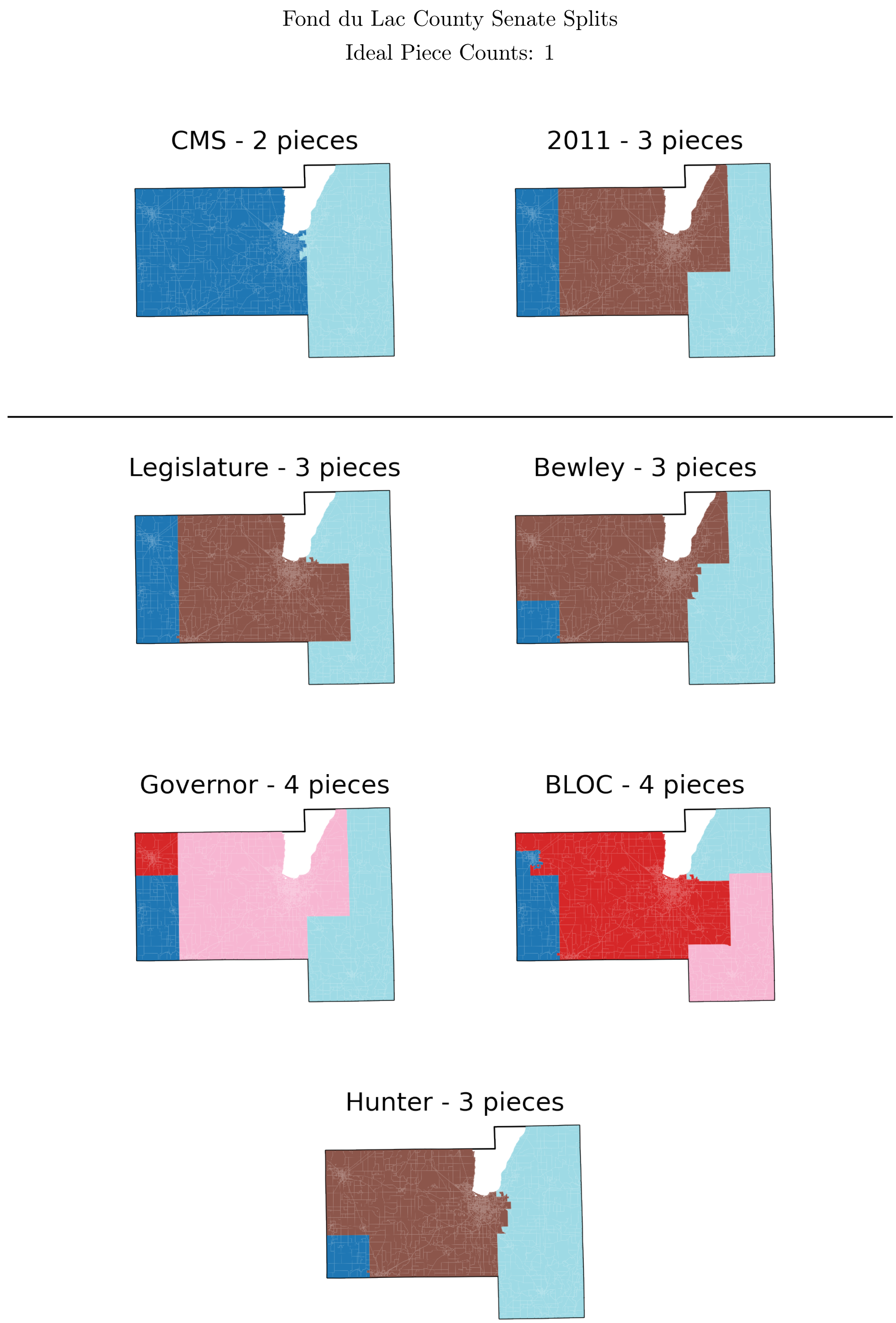


Figure 17: Assembly Splits of Green County

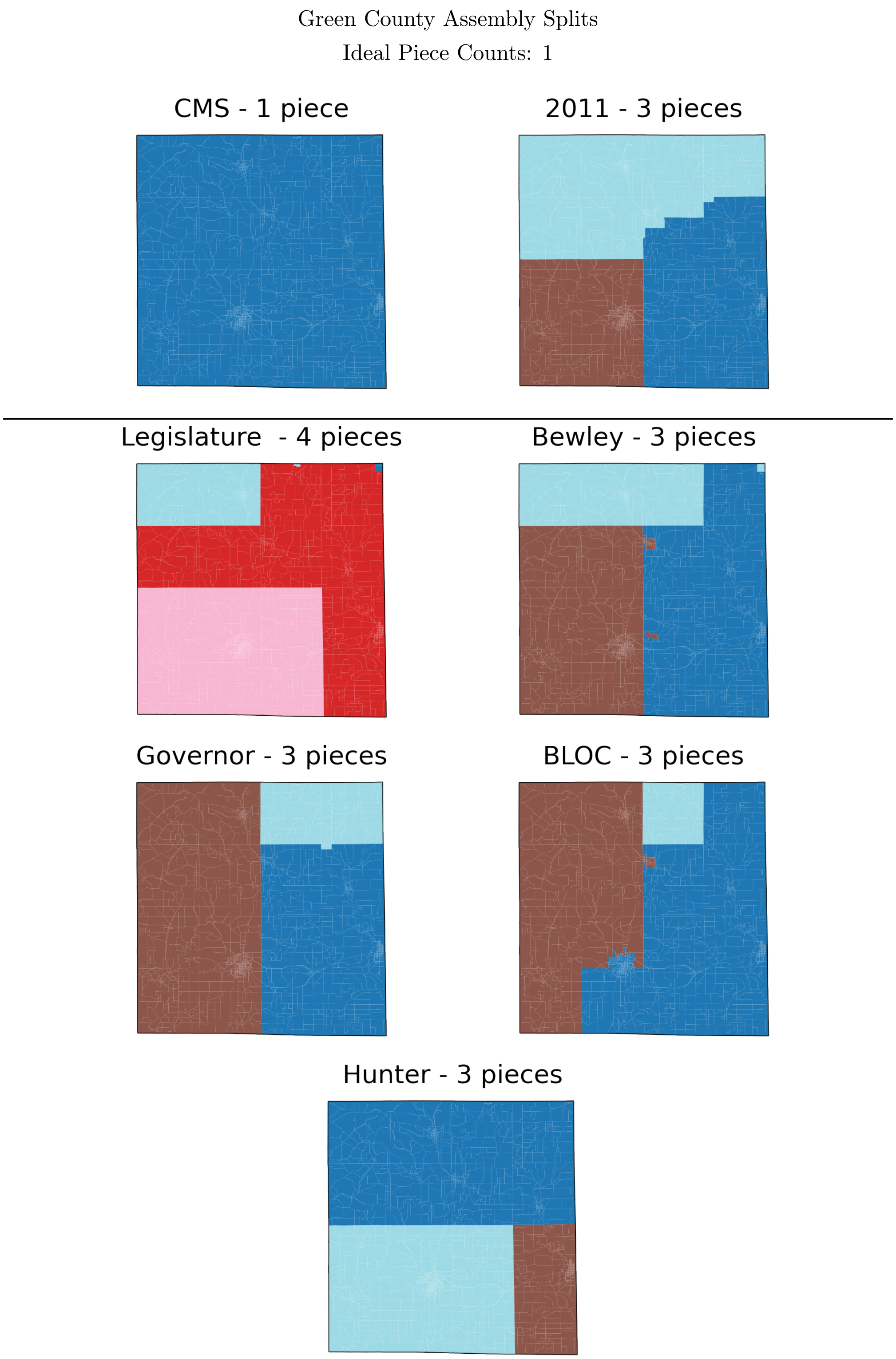


Figure 18: Senate Splits of Green County

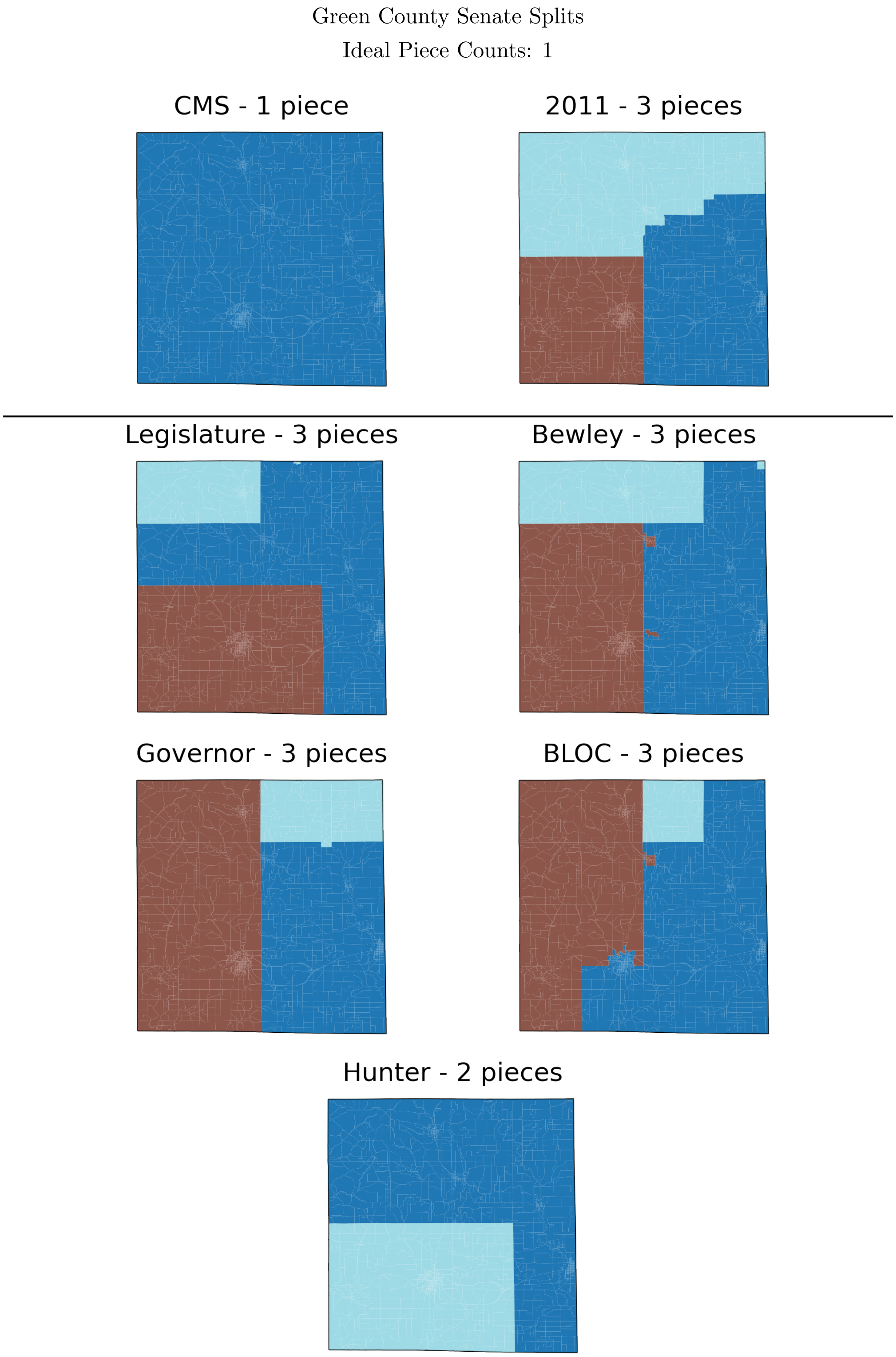


Figure 19: Assembly Splits of Shawano County

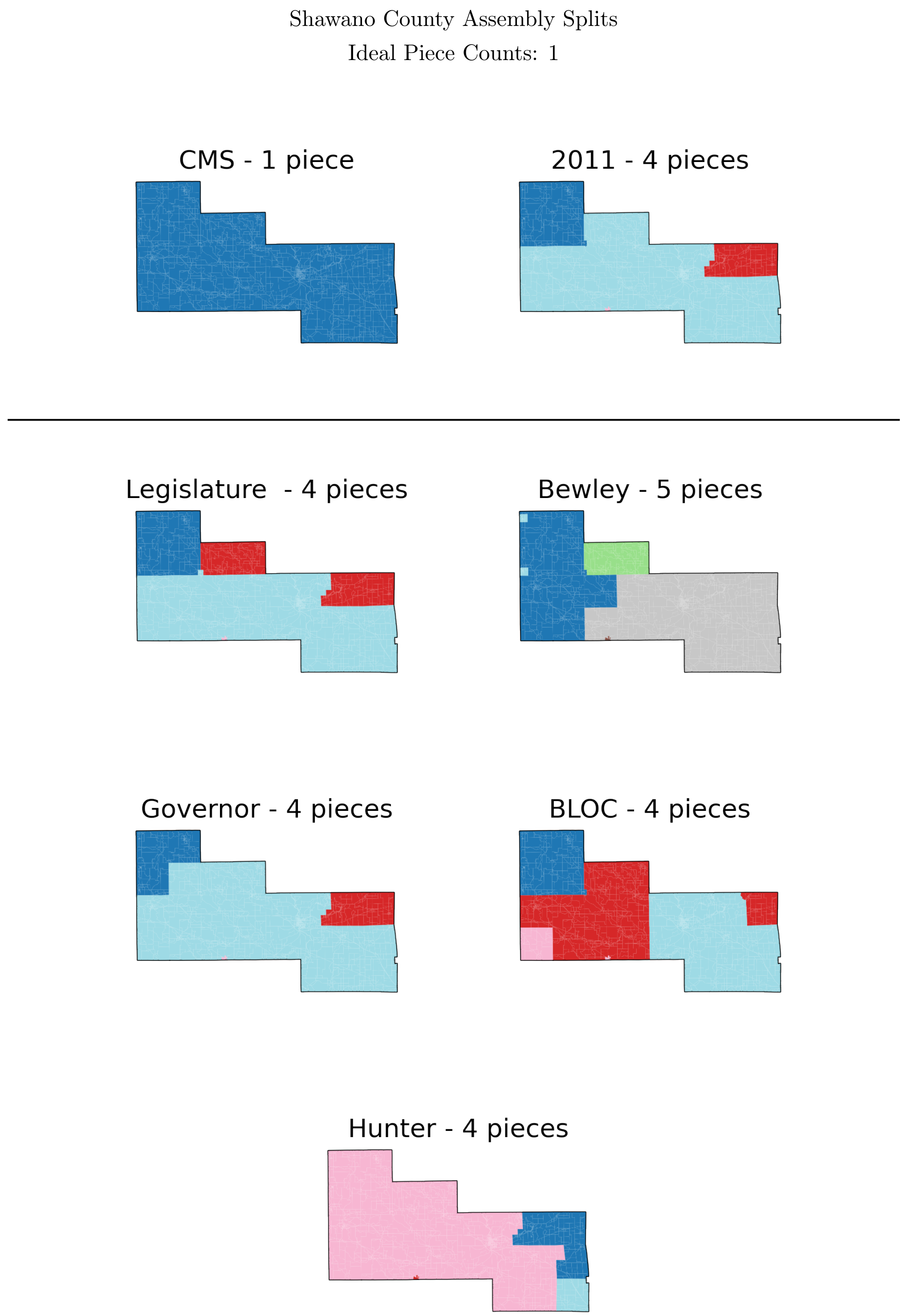


Figure 20: Senate Splits of Shawano County

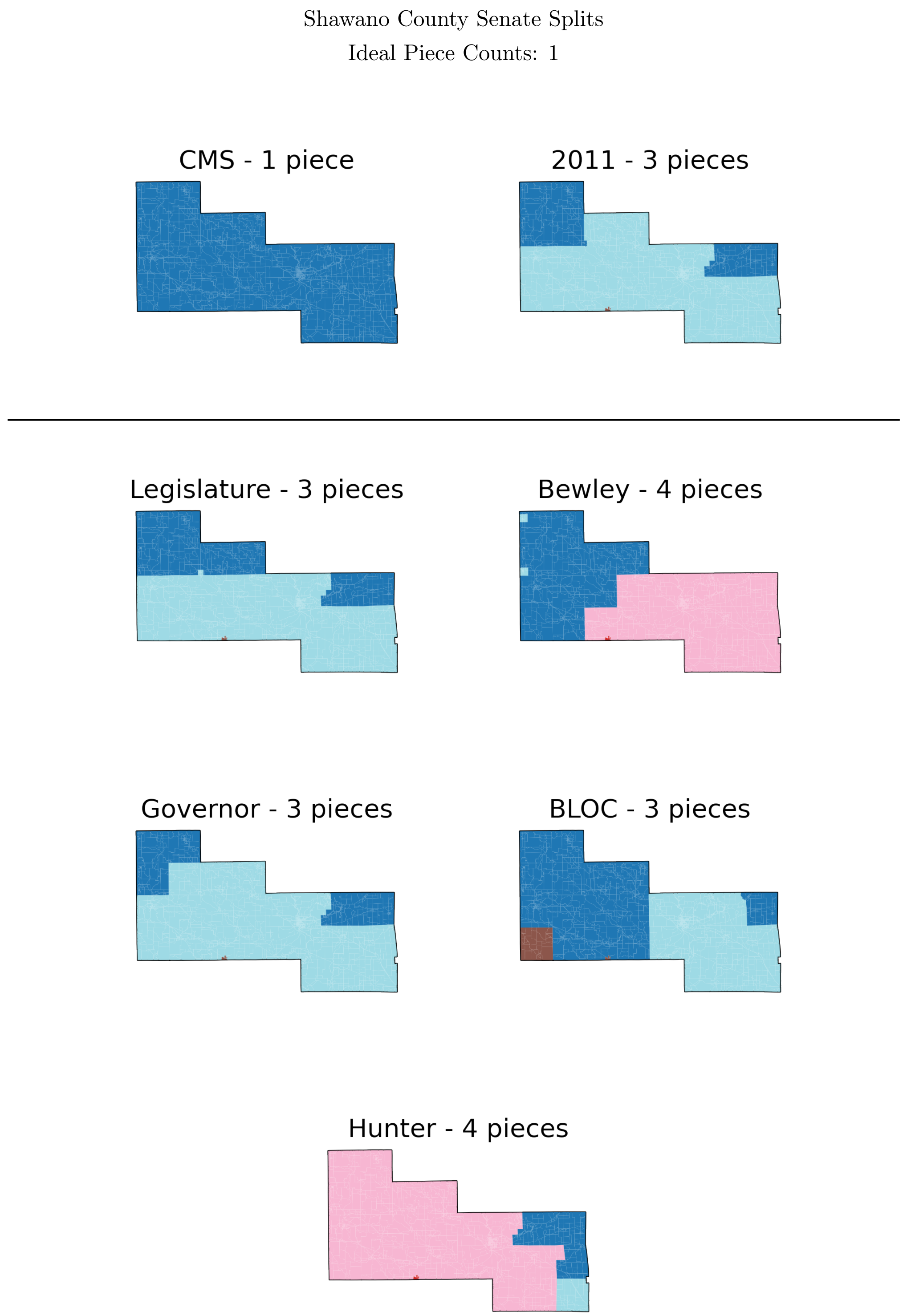


Figure 21: Assembly Splits of Winnebago County

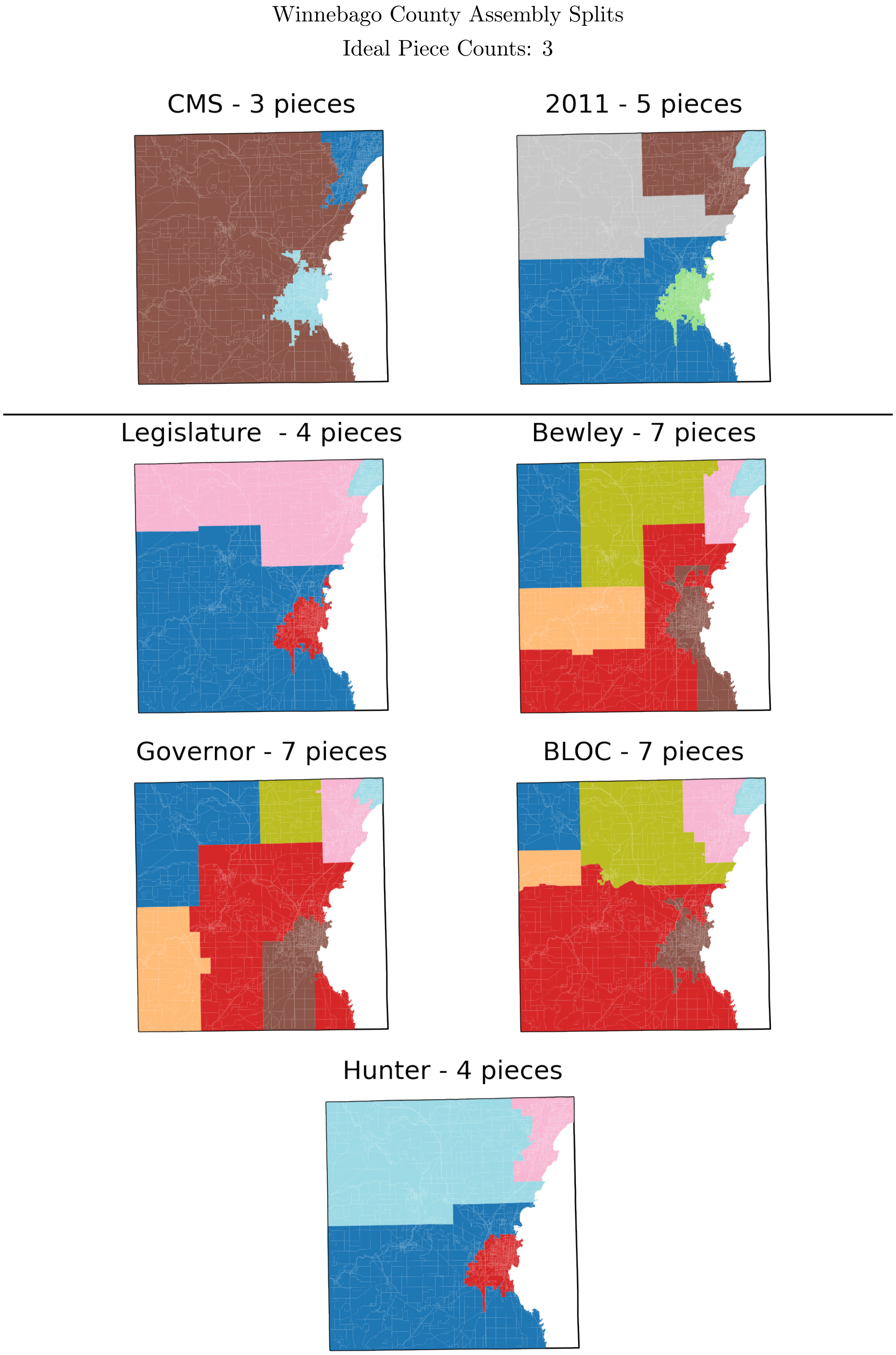
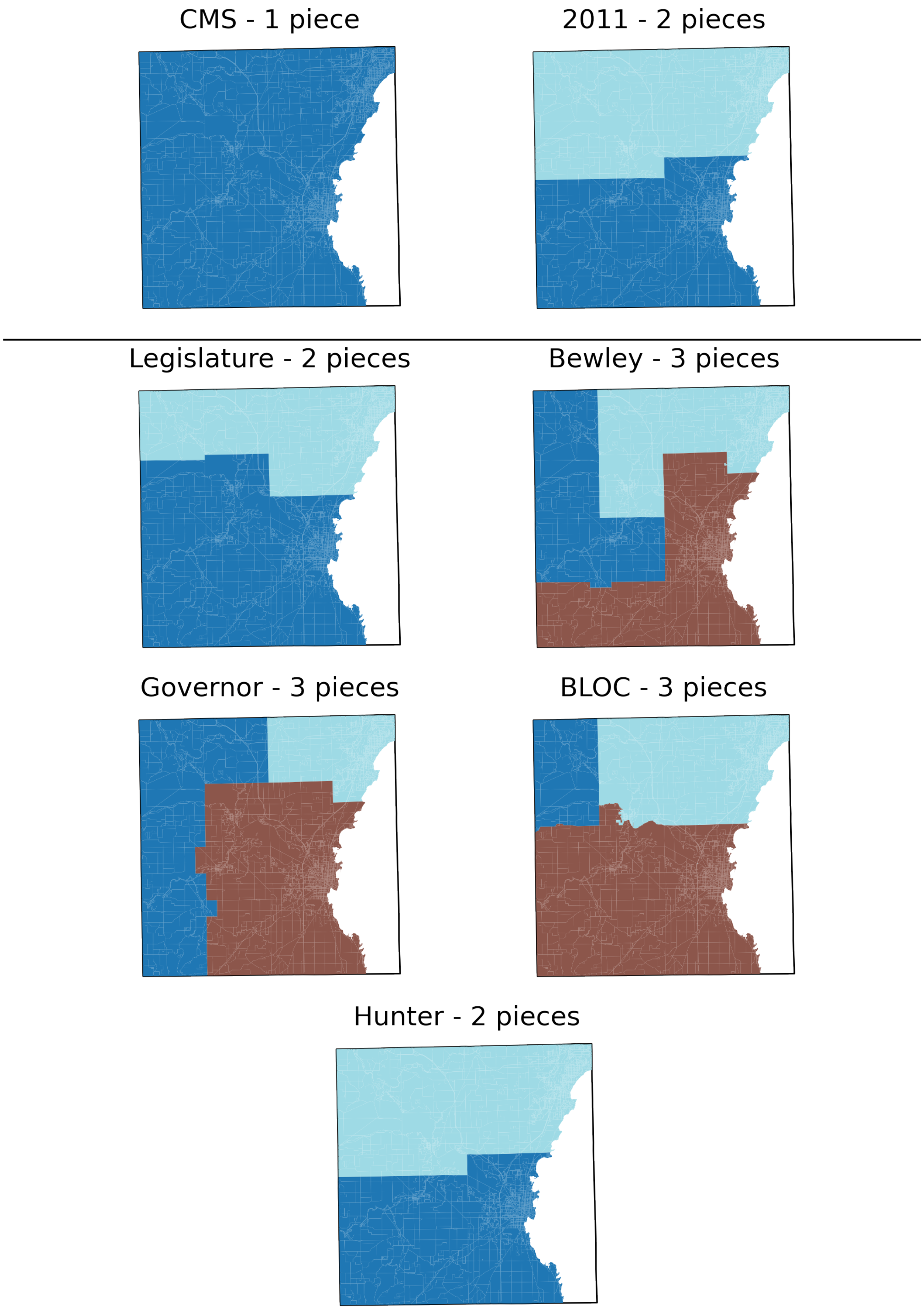


Figure 22: Senate Splits of Winnebago County

Winnebago County Senate Splits
Ideal Piece Counts: 1



As these figures and the above discussion illustrate, the CMS plans necessarily make modifications to district boundaries to achieve the lowest population deviation, fewest county splits, and extremely competitive compactness scores. Another metric confirms that parties are wrong to disregard the CMS state legislative plans because of their performance on core retention. This metric measures the percentage of internal district boundaries[9] that do not correspond to a district boundary in the 2011 map or an existing county line. Results from this measurement are presented in Table 1 below, demonstrating that the CMS plans behave similarly to the other plans. This metric also incorporates compactness, in the form of the perimeter of the districts, which is a commonly used metric for evaluating compactness of districting plans.

Table 1: Percentage Overlap with Prior District or County Lines

Plan Name	CMS	SB621	BEWLEY	BLOC	GOV	HUNTER
ASM Enacted or County Boundary Proportion (%)	69.5	71.2	63.5	72.1	77.9	70.0
SEN Enacted or County Boundary Proportion (%)	77.4	74.0	62.1	69.3	77.2	63.6

IV.F State Legislative Conclusion

In my First Report, I evaluated alternatives to the CMS state legislative plans and concluded that all fell short of the CMS plans, which makes necessary modifications to district boundaries to achieve lower population, fewer county splits, and extremely competitive compactness scores, among other positive attributes. After reviewing the responsive materials filed on December 30, 2021, nothing causes me to alter that conclusion. While several parties tout their performance on particular metrics, the CMS maps navigate the tradeoffs between them, achieving the best or near-best result on each.

V Analysis of Congressional Plans

In my First Report, I analyzed the performance of alternatives to the CMS congressional map on a number of criteria germane to congressional redistricting in Wisconsin, including population deviation, VRA compliance, least change, respect for counties, towns, and wards, and compactness. Based on that review, I was not able to identify any plan that performs as effectively under the applicable framework.

Counsel for CMS subsequently asked me to review the responsive briefs and expert reports submitted by the proponents of each alternative to the CMS congressional plan. For the reasons set out below, the arguments and analyses contained in the December 30, 2021 submissions do not alter my conclusion that the CMS congressional map should be selected. I do not address population deviation or VRA compliance because no party disputes that the CMS map achieves optimal performance on population deviation or VRA compliance.

V.A Least Change

At least one party asserts that the CMS map should not be selected because it moves more people than other maps. Governor's Response Br. 23. I am not aware of any quantitative threshold that should be applied in evaluating this metric of least change and note that the proposed maps are all within approximately three percentage points of each other on this metric. Moreover, the parties' responses ignore that the CMS plan performs well on several additional measures of least change. *See* Governor's Response Br. 22-23.

For example, as illustrated in Table 3 in my First Report, the CMS map performs well on the preserved edges measure, which means that it preserves many pairs of adjacent census blocks from the 2011 enacted plan. It also performs as well as other maps on district and county overlap, and performs well on the retention of people and area. One party also criticizes the least change credentials of the CMS congressional plan on the basis that it does not comport with Wisconsin's political geography. *See* Congressmen's Br.

[9] This computation does not include the external boundary of the state, which is the same for all maps.

20. However, if leaving political subdivisions like counties and municipalities is probative on this question, *see* Congressmen Br. 20 (declining to discuss counties but noting that preservation of municipal lines is "probative as to whether district respects Wisconsin's political geography"), then then the CMS plan in fact comports very well with Wisconsin political geography. As Table 4 of my First Report demonstrates, the CMS Congressional plan protects more county lines than any alternative map. Figure 23 illustrates just one example of a county protected by the CMS Congressional plan, but split by alternative proposals. Figures 24 and 25 extend this observation, providing examples where the CMS Congressional plan makes whole counties that were divided in the 2011 enacted Congressional plan and remain divided in some or all alternatives to the CMS plan.

Figure 23: Congressional Splits of Sauk County

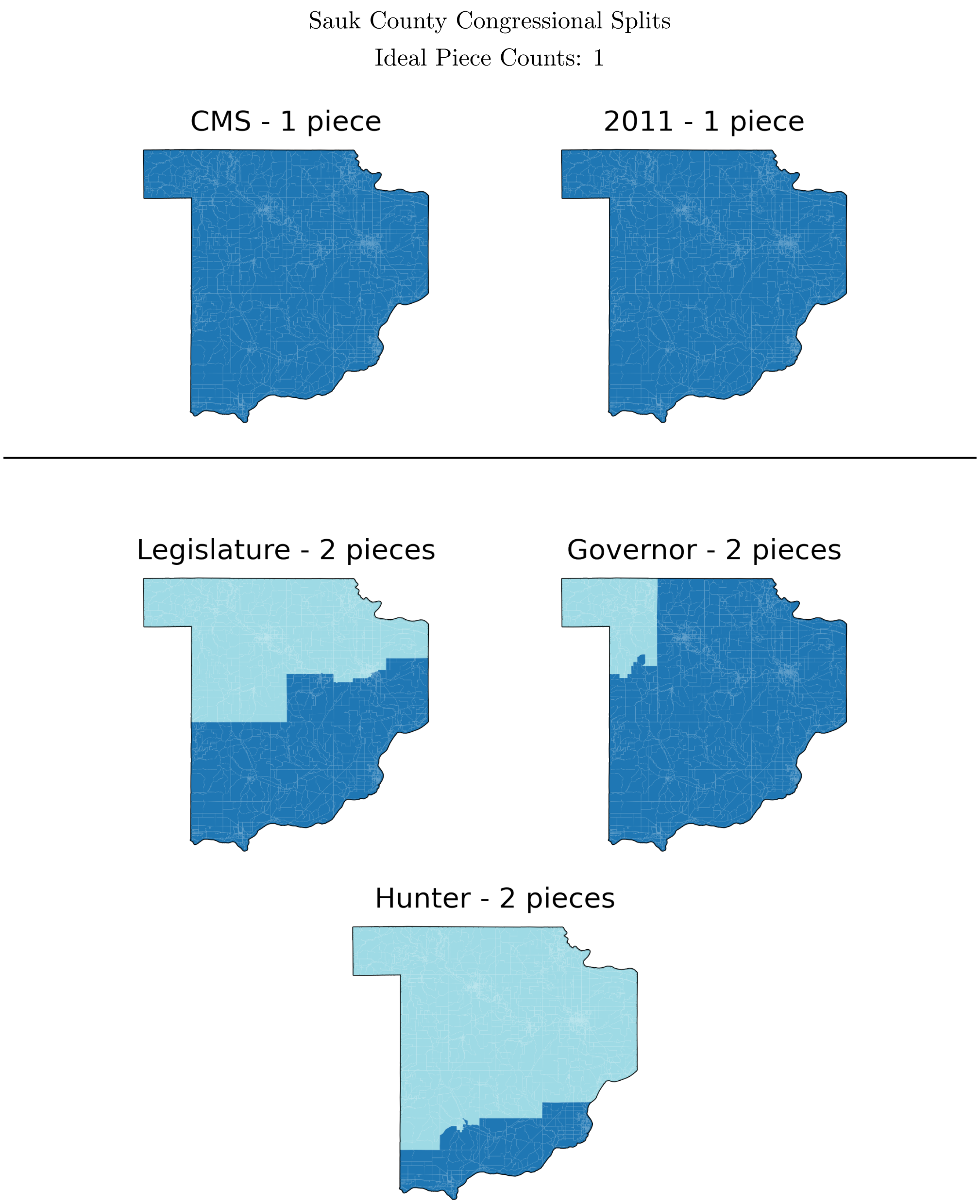


Figure 24: Congressional Splits of Walworth County

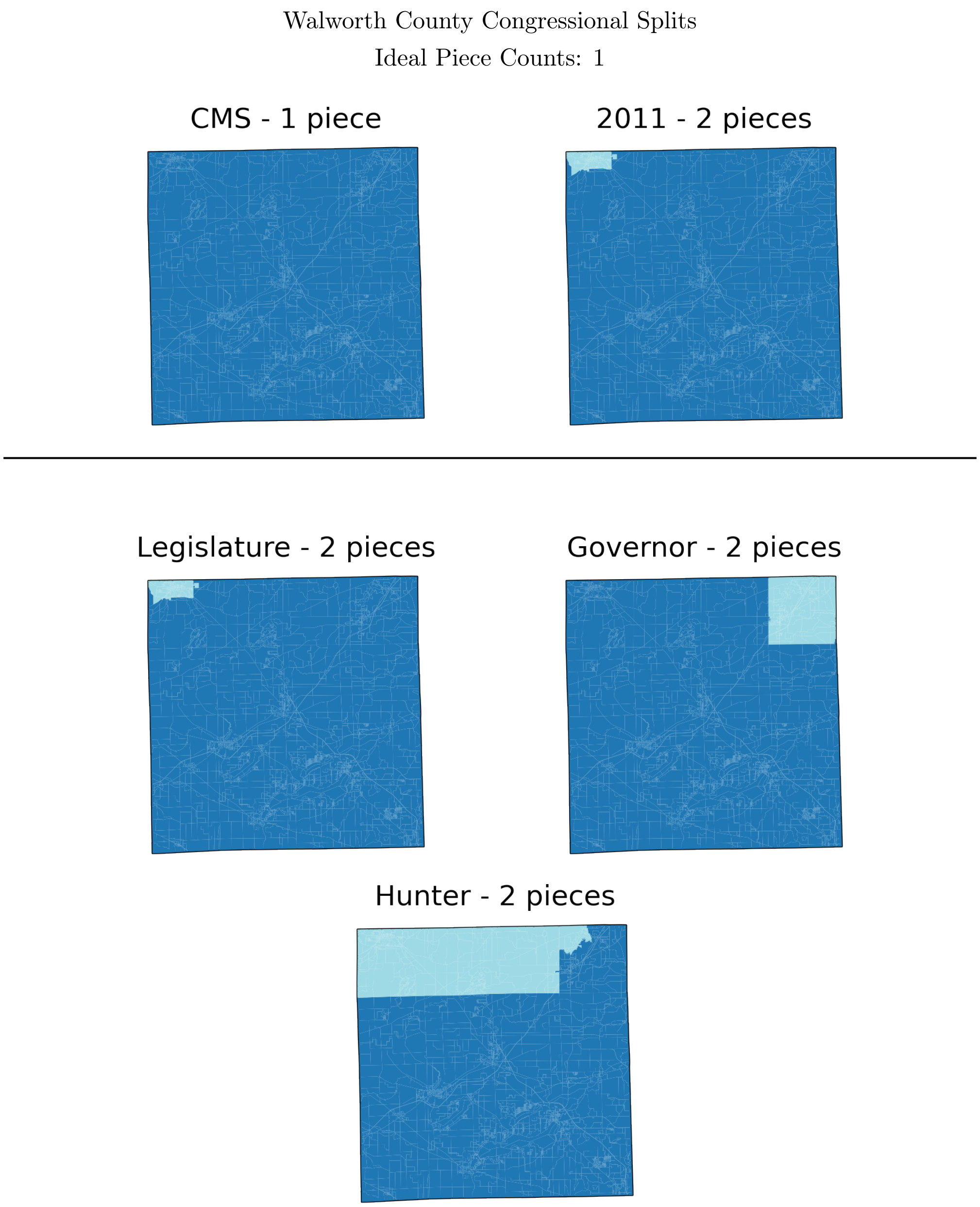
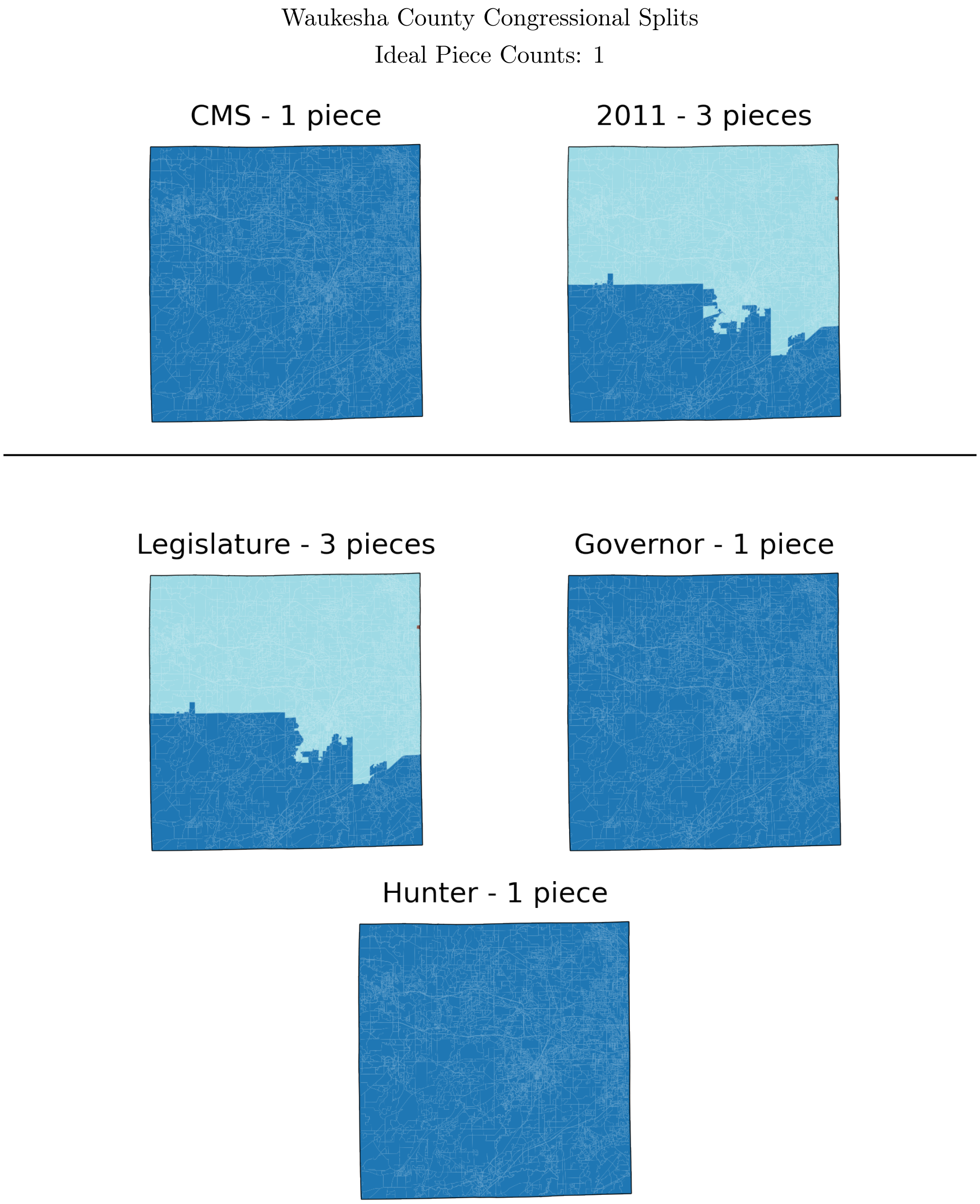


Figure 25: Congressional Splits of Waukesha County



As with the state legislative maps, it is also useful to measure the percentage of district boundaries that correspond to either a district boundary in the 2011 map or an existing county line to evaluate the least change credentials of the proposed maps. That metric is presented in Table 2 below, and demonstrates that CMS plan is very competitive under this measure, which relates the county protection and compactness criteria to least change directly.

Table 2: Percentage Overlap With Prior District or County Lines

Plan Name	CMS	SB621	GOV	HUNTER
CON Enacted or County Boundary Proportion (%)	87.5	79.5	86.1	88.8

V.B Congressional Summary

For these reasons, the December 30 submissions do not disturb my conclusion that the CMS congressional plan performs most effectively under the applicable framework.

I declare under penalty of perjury of the United States that the foregoing is true and correct to the best of my knowledge and understanding.

Dated: January 4, 2022

Daryl R. Deford

Appendix A

DARYL R. DEFORD

Curriculum Vitae

328 Neill Hall WSU Pullman, WA ⋄ (509) 205–7347

daryl.deford@wsu.edu ⋄ daryldeford.com

ACADEMIC APPOINTMENTS

Washington State University, Pullman, WA *August 2020 – Present*
Assistant Professor of Data Analytics – Department of Mathematics and Statistics

Massachusetts Institute of Technology, Cambridge, MA *June 2018 – July 2020*
Postdoctoral Associate – CSAIL Geometric Data Processing Group
Advisor: Justin Solomon

Tufts University, Medford, MA *June 2018 – July 2020*
Visiting Scholar – Jonathan M. Tisch College of Civic Life
Advisor: Moon Duchin

EDUCATION

Dartmouth College, Hanover, NH *September 2013 – June 2018*
Ph.D. Mathematics *Awarded June 2018*
Advisor: Dan Rockmore
Dissertation: Matched Products and Dynamical Models for Multiplex Networks
A.M. Mathematics *Awarded November 2014*

Washington State University, Pullman, WA *August 2010 – May 2013*
B.S. in Theoretical Mathematics *Awarded May 2013*
Summa Cum Laude

RESEARCH PUBLICATIONS

Accepted Papers

A24: *Random Walks and the Universe of Districting Plans* (with M. Duchin), Book Chapter in *Political Geography*, Birkhäuser, to appear 2022.

A23: *Implementing Partisan Symmetry: Problems and Paradoxes* (with N. Dhamankar, M. Duchin, V. Gupta, M. McPike, G. Schoenbach, K. W. Sim), Political Analysis, arxiv: 2008.06930, to appear 2022.

A22: *Empirical Sampling of Connected Graph Partitions for Redistricting* (with L. Najt and J. Solomon), Physical Review E, 104(6), 064130, 2021.

A21: *Partisan Dislocation: A Precinct-Level Measure of Representation and Gerrymandering* (with N. Eubank and J. Rodden), Political Analysis, 1-23, doi:10.1017/pan.2021.13, 2021.

A20: *Colorado in Context: Congressional Redistricting and Competing Fairness Criteria in Colorado* (with J. Clelland, H. Colgate, B. Malmskog, and F. Sancier-Barbosa), Journal of Computational Social Science, doi:10.1007/s42001-021-00119-7, 2021.

A19: *ReCombination: A family of Markov chains for redistricting* (with M. Duchin and J. Solomon), Harvard Data Science Review, 3(1), 2021.

A18: *Medial Axis Isoperimetric Profiles* (with J. Solomon and P. Zhang), Computer Graphics Forum, 39(5), 1-13, 2020.

A17: *On the Spectrum of Finite, Rooted Homogeneous Trees* (with D. Rockmore), Linear Algebra and its Applications, 598, 165-185, 2020.

- A16: *Competitiveness Measures for Evaluating Districting Plans* (with M. Duchin and J. Solomon), Statistics and Public Policy, 7(1), 69-86, 2020.
- A15: *Mathematics of Nested Districts: The Case of Alaska* (with S. Caldera, M. Duchin, S. Gutenkust, and C. Nix), Statistics and Public Policy, 7(1), 39-51, 2020.
- A14: *Aftermath: The ensemble approach to political redistricting* (with J. Clelland and M. Duchin), MAA Math Horizons, 28(1), 34-35, 2020.
- A13: *Total Variation Isoperimetric Profiles* (with H. Lavenant, Z. Schutzman, and J. Solomon), SIAM J. Appl. Algebra Geometry, 3(4), 585-613, 2019.
- A12: *Spectral Clustering Methods for Multiplex Networks* (with S. Pauls) Physica A: Statistical Mechanics and its Applications, 533, 121949, 2019.
- A11: *Redistricting Reform in Virginia: Districting Criteria in Context* (with M. Duchin), Virginia Policy Review, 12(2), 120-146, 2019.
- A10: *A New Framework for Dynamical Models on Multiplex Networks* (with S. Pauls), Journal of Complex Networks, 6(3), 353-381, 2018.
- A9: *Cyclic Groups with the same Hodge Series*, (with P. Doyle), Revista de la Uniòn Matemática Argentina, 59(2), 241–254, 2018.
- A8: *Multiplex Dynamics on the World Trade Web*, Proc. 6th International Conference on Complex Networks and Applications, Studies in Computational Intelligence, Springer, 1111–1123, 2018.
- A7: *Random Walk Null Models for Time Series Data*, (with K. Moore), Entropy, 19(11), 615, 2017.
- A6: *Enumerating Tilings of Rectangles By Squares*, Journal of Combinatorics, 6(3), 339-351, 2015.
- A5: *Enumerating Distinct Chessboard Tilings*, Fibonacci Quarterly, 52(5), 102-116, 2014.
- A4: *Pulsated Fibonacci Sequences* (with K. Atanassov and A. Shannon), Fibonacci Quarterly, 52(5), 22-27, 2014.
- A3: *Seating Rearrangements on Arbitrary Graphs*, Involve: A Journal of Mathematics, 7(6), 787-805, 2014.
- A2: *Empirical Analysis of Space-Filling Curves for Scientific Computing Applications* (With A. Kalyanaraman), Proc. 42nd International Conference on Parallel Processing, 170-179, 2013.
- A1: *Counting Rearrangements on Generalized Wheel Graphs*, Fibonacci Quarterly, 51(3), 259-273, 2013.

Preprints

- P4: *Bayesian Inference of Random Dot Product Graphs via Conic Programming* (with D. Wu and D. Palmer), arXiv:2101.02180.
- P3: *Complexity and Geometry of Sampling Connected Graph Partitions* (with L. Najt and J. Solomon), arXiv: 1908.08881.
- P2: *Fourier Transforms on $SL_2(\mathbb{Z}/p^n\mathbb{Z})$ and Related Numerical Experiments* (with B. Breen, J. Linehan, and D. Rockmore), arXiv:1710.02687.
- P1: *A Random Dot Product Model for Weighted Networks* (with D. Rockmore) arXiv: 1611.02530.

Technical Reports

- T6: *Ensemble Analysis for 2021 Legislative Redistricting in Colorado, First and Second Staff Plans* (with J. Clelland, B. Malmskog, and F. Sancier-Barbosa), Colorado in Context Report, 2021.
- T5: *Ensemble Analysis for 2021 Congressional Redistricting in Colorado* (with J. Clelland, B. Malmskog, and F. Sancier-Barbosa), Colorado in Context Report, 2021.
- T4: *Comparison of Districting Plans for the Virginia House of Delegates* (with M. Duchin and J. Solomon), MGGG Technical Report, 2019.
- T3: *Amicus Brief of Mathematicians, Law Professors, and Students* (with M. Duchin and G. Charles et al.), Rucho v. Common Cause, Supreme Court, 2019.
- T2: *Study of Reform Proposals for Chicago City Council* (with M. Duchin et al.), MGGG Technical Report, 2019.
- T1: *An Application of the Permanent–Determinant Method: Computing the Z-Index of Arbitrary Trees*, WSU Department of Mathematics Technical Report Series 2013 #2, 2013.

TEACHING EXPERIENCE

Washington State University Pullman, WA
Instructor *Fall 2020 - Present*

- Designed syllabi and daily lectures. Wrote and graded homework, quizzes, and exams. Fully responsible for course content and material.

Math 448/548 - Numerical Analysis *Spring 2022*

Fundamental course on numerical computation, including: finding zeroes of functions, approximation and interpolation, numerical integration, numerical solution of ordinary differential equations, and numerical linear algebra.

STAT 419 - Introduction to Multivariate Statistics *Fall 2021*

Introductory course covering multidimensional data, multivariate normal distribution, principal components, factor analysis, clustering, and discriminant analysis.

Data 115 - Introduction to Data Analytics *Fall 2020, 2021 Spring 2021*

Basic techniques and methodology of data science, with an emphasis on data processing and software tools. This course provides a foundation for beginning data analytics majors as well as students from across the university who are looking to develop data and quantitative literacy.

Math 581 - Topics in Math (Computational Methods in Complex Networks) *Fall 2020*

Introduction to computational methods and software for analyzing complex systems as well as applications of partition sampling to political redistricting.

Metric Geometry and Gerrymandering Group Cambridge, MA
VRDI Instructor *Summer 2018, 2019*

- Organized and led student research groups during an eight week summer program on political redistricting for 80+ graduate and undergraduate students. Met with students daily and both generated and supervised a wide variety of research projects in computational, mathematical, and political topics.

Tufts University Medford, MA
Co-Instructor *Spring 2019*

- Co–taught STS 10: Reading Lab on Mathematical Models in Social Context. This is a reading and discussion based course focused on providing an STS perspective to students who are taking technically-focused modeling classes.

Massachusetts Institute of Technology Cambridge, MA
IAP Instructor *January 2019*

- Developed a four-week course on computational methods for political redistricting. The course incorporated cutting edge mathematical and computational techniques for analyzing gerrymandering.

Dartmouth College Hanover, NH
Instructor *September 2015 - May 2018*

- Designed syllabi and daily lectures. Wrote and graded homework, quizzes, and exams. Fully responsible for course content and material.

Math 36/QSS 36 - Mathematical Modeling in the Social Sciences *Fall 2017*

Data driven course exploring mathematical models and analysis techniques

UNSG 100 - Graduate Ethics Seminar *Fall 2017, 2016, 2015*

Seminar on ethical and professional issues in science and mathematics

Math 8 - Calculus of Functions of one and Several Variables *Winter 2017*

Second term calculus course covering infinite series, vector functions, and partial derivatives

Math 1 - Calculus with Algebra *Fall 2015*

Introductory calculus course with an emphasis on limits and differentiation

Teaching Assistant — *September 2013 - June 2015*

- Held tutorial sessions three times per week. Graded quizzes and exams. Designed computing assignments and tutorials for linear algebra.

  Math 23 - Differential Equations — *Spring 2015*
  Math 22 - Linear Algebra with Applications — *Fall 2014*
  Math 3 - Calculus — *Winter 2014*
  Math 12 - Calculus Plus — *Fall 2013*

Washington State University — Pullman, WA
Undergraduate Teaching Assistant — *August 2012 - May 2013*

- Held tutorial sessions and graded homework and exams. Supervised a mathematical computing lab.

  Math 320 - Modern Algebra — *Spring 2013*
  Math 330 - Secondary Teaching — *Spring 2013*
  Math 315 - Differential Equations — *Fall 2012*

EDUCATIONAL OUTREACH

UW Data Science for Social Good — Seattle, WA
Project Lead — *Summer 2021*

- Designed and supervised a research project for four data science fellows on applications of ensemble methods to initial districting plan evaluation. The fellows gave a public presentation of their work and developed a user guide "Applying GerryChain: A Users Guide for Redistricting Problems" with accompanying website, case studies, and code examples to demonstrate good modeling practives and support other researchers working on these problems.

New Hampshire State Math Team — Manchester, NH
Math Team Coach — *Fall 2018–2020*

- Designed practice problems and preparatory exercises for the AMC exams, ARML, MMATH, and HMMT. Led monthly problem solving sessions and group activities.

LaTeX Workshops — Hanover, NH
Organizer — *Fall 2016–May 2018*

- Designed and presented a series of eleven one hour–long and two three hour–long workshops on mathematical typesetting in LaTeX with D. Freund and K. Harding. Resources and lesson plans

Crossroads Academy Math Team — Lyme, NH
Math Team Coach — *September 2015 – May 2018*

- Designed practice problems and preparatory exercises for the AMC exams, MathCounts, and MathLeague. Led weekly problem solving sessions and group activities. During 2015–17, the Crossroads team twice won the Chapter and State MathCounts and MathLeague competitions and placed first in Northern New England on the AMC-8.

New Hampshire State MathCounts Team — Lyme, NH
Math Team Coach — *March 2017 – May 2017*

- Designed practice problems and preparatory exercises for the national MathCounts exam. Led biweekly problem solving sessions and group activities. Students competed in the national competition in Orlando, Florida.

Johns Hopkins Center for Talented Youth Science and Technology Series Hanover, NH
Workshop Leader

· Developed and presented hour–long workshops for high school students.

Modern Cryptography (with D. Freund) *October 2014*
Forensic Accounting *April 2016*
Binary and Barcodes (with D. Freund) *April 2017*

Dartmouth College Exploring Mathematics Camp Hanover, NH
Co-Instructor

· Organized and presented week long math camps for high school students.
Mathematics of Games *August 2015*
Cryptography *July 2015*

RESEARCH PRESENTATIONS

Talks

1. Analysis Seminar, Pullman, WA *December 2021*
   Introduction to Graphons I and II
2. PPPA Research Colloquium, Pullman, WA *November 2021*
   Computational Methods for Evaluating Districting Plans
3. INFORMS Annual Meeting, Zoom *October 2021*
   Algorithms And Analysis For Centered Redistricting Plans
4. WSU Math Club, Pullman, WA *October 2021*
   Graphs, Geometry, and Gerrymandering
5. Civic Hackathon, Madison, WI *September 2021*
   Introduction to Computational Redistricting
6. Harvard Redistricting Algorithms, Law, and Policy Cambridge, MA *September 2021*
   Technical State of the Art for Computational Redistricting
7. ASA Joint Statistical Meeting, Zoom *August 2021*
   Computational Methods for Assessing Political Redistricting Reforms
8. New Mexico Redistricting Commission, Santa Fe, NM *July 2021*
   Markov chain ensemble metrics for evaluation of redistricting plans
9. Colorado College Summer Program, Colorado Springs, CO *June 2021*
   Computational Redistricting Analysis
10. WSU Seminar in Statistics, Pullman, WA *April 2021*
    Ensemble Analysis for the 2020 Redistricting Cycle
11. Princeton Gerrymandering Project, Princeton, NJ *March 2021*
    Computational Redistricting in 2021
12. Combinatorics, Linear Algebra, and Number Theory, WSU, Pullman, WA *March 2021*
    Gerry-Matchings and Pair-y-Mandering
13. JMM 2021, Washington DC *January 2021*
    Short Course: Mathematical and Computational Methods for Complex Social Systems
14. INFORMS Special Session on Fairness in Operations Research, Baltimore, MD *November 2020*
    Computational Methods For Assessing Districting Plans
15. WSU Seminar in Statistics, Pullman, WA *November 2020*
    Statistical and Computational Methods for Assessing Political Redistricting
16. Pi MU Epsilon Lecture, St. Michael's College, Colchester, VT *October 2020*
    Graphs, Geometry, and Gerrymandering
17. ADSA Annual Meeting, Zoom *October 2020*
    Geospatial Data for Political Redistricting Analysis

18. Common Experience Lecture, Texas State University, San Marcos, TX *October 2020*
Graphs, Geometry, and Gerrymandering
19. Combinatorics, Linear Algebra, and Number Theory, WSU, Pullman, WA *September 2020*
Representations of $SL_2(\mathbb{Z}/p^n\mathbb{Z})$ and spectral properties of Bethe trees
20. CGAD-GTOpt Seminar, Washington State University, Pullman, WA, *July 2020*
Geometric and Optimization Problems Motivated by Political Redistricting
21. Redistricting Conference 2020, Duke University, Durham, NC, *March 2020*
Multiresolution Redistricting Algorithms
22. Math Department Colloquium, College of Charleston, Charleston, SC. *February 2020*
Geospatial Data, Markov Chains, and Political Redistricting
23. Math Department Colloquium, Washington State University, Pullman, WA. *January 2020*
Geospatial Data, Markov Chains, and Political Redistricting
24. JMM 2020, Denver, CO. *January 2020*
Markov chains for sampling connected graph partitions
25. Math Department Colloquium, Pacific University, Forest Grove, OR. *January 2020*
The Mathematics of Nested Legislative Districts
26. MIT Graphics Annual Retreat, North Falmouth, MA. *October 2019*
Connected Graph Partitions and Political Districting
27. Topology, Geometry and Data Seminar, Ohio State University, Columbus, OH. *September 2019*
Hardness results for sampling connected graph partitions with applications to redistricting
28. Math Department Colloquium, Denison University, Granville, OH. *September 2019*
Graphs, Geometry, and Gerrymandering
29. Math Department Colloquium, Oberlin College, Oberlin, OH. *September 2019*
Graphs, Geometry, and Gerrymandering
30. Math Department Colloquium, College of Wooster, Wooster, OH. *September 2019*
Graphs, Geometry, and Gerrymandering
31. Math Monday Colloquium, Kenyon College, Gambier, OH. *September 2019*
Graphs, Geometry, and Gerrymandering
32. Applied Math Seminar, University of Massachusetts Lowell, Lowell, MA. *September 2019*
Hardness results for sampling connected graph partitions with applications to redistricting
33. Math Department Colloquium, Yale University, New Haven, CT. *August 2019*
Mathematical Challenges in Neutral Redistricting
34. Voting Rights Data Institute Seminar, Cambridge, MA. *June 2019*
A Friendly Introduction to Discrete MCMC
35. Voting Rights Data Institute Seminar, Cambridge, MA. *June 2019*
Graphs and Networks: Discrete Approaches to Redistricting
36. Math Department Colloquium, Dartmouth College, Hanover, NH. *April 2019*
Total Variation Isoperimetric Profiles and Political Redistricting
37. ACM Seminar, Dartmouth College, Hanover, NH. *April 2019*
Hardness results for sampling connected graph partitions with applications to redistricting
38. Unrig Summit Masterclass, Nashville, TN. *March 2019*
Legal and Math Deep Dive: Gerrymandering and Redistricting
39. MIT Graphics Seminar, Cambridge, MA. *March 2019*
Computational Challenges in Neutral Redistricting
40. JMM 2019, Baltimore, MD. *January 2019*
Matched Products and Stirling Numbers of Graphs
41. Societal Concerns in Algorithsm and Data Analysis, Weizmann Institute of Science, Rehovot, Israel. *December 2018*
Computational Problems in Neutral Redistricting
42. Math and Law of Redistricting, Radcliffe Institute, Cambridge, MA. *December 2018*
GerryChain and MCMC tutorials
43. Math Colloquium, Tufts University, Medford, MA. *November 2018*
Matched Products and Stirling Numbers of Graphs

44. MIT Graphics Annual Retreat, Dedham, MA. *October 2018*
Mathematical Challenges in Neutral Redistricting
45. SAMSI Workshop on Quantitative Redistricting, Duke University, Durham, NC. *October 2018*
Compactness Profiles and Reversible Sampling Methods for Plane and Graph Partitions
46. Election Teach–in, SMFA, Boston, MA. *October 2018*
Computational Challenges in Political Redistricting
47. STS Seminar, Tufts University, Cambridge, MA. *September 2018*
Mathematical Modeling of Social Connections
48. Voting Rights Data Institute Seminar, Cambridge, MA. *June 2018*
Introduction to Monte Carlo Methods
49. Mathematics Colloquium, University of Central Florida, Orlando, FL. *February 2018*
Dynamical Models for Multiplex Data
50. Mathematics Colloquium GVSU, Grand Valley, MI. *February 2018*
Random Walk Null Models for Time Series
51. Omidyar Fellowship Presentation, Santa Fe, NM. *January 2018*
Mathematical Embeddings of Complex Systems
52. Mathematics Colloquium at University of San Fransisco, San Fransisco, CA. *January 2018*
Dynamical Models for Multiplex Data
53. Mathematics Colloquium at Providence College, Providence, RI. *January 2018*
Dynamical Models for Multiplex Data
54. JMM, San Diego, CA. *January 2018*
Dynamical Modeling for Multiplex Networks
55. International Complex Networks Conference Lyon, France. *December 2017*
Multiplex Dynamics on the World Trade Web
56. Physics Colloquium at Washington University, St. Louis, MO. *October 2017*
Spectral Clustering on Multiplex Data
57. SIAM Annual Meeting, Pittsburgh, PA. *July 2017*
Permutation Complexity Measures for Time Series
58. Applied and Computational Mathematics Seminar, Hanover NH. *November 2016*
Random Dot Product Models for Weighted Networks
59. Inference on Networks: Algorithms, Phase Transitions, New Models and New Data, Santa Fe, NM. *December 2015*
Dynamically Motivated Models for Multiplex Networks
60. Applied Math Days, Troy, NY. *April 2015*
Multiplex Structure on the World Trade Web
61. Graduate Student Combinatorics Conference, Lexington, KY. *March 2015*
Total Dynamics on Multiplex Networks
62. Sixteenth International Fibonacci Conference, Rochester, NY. *July 2014*
Enumerating Distinct Chessboard Tilings
63. Dartmouth Graduate Student Seminar, Hanover, NH. *(Quarterly) 2013 - 2018*
Various Topics
64. Joint Mathematics Meeting, San Diego, CA. *January 2013*
Counting Combinatorial Rearrangements, Tilings with Squares and Symmetric Tilings
65. West Coast Number Theory Conference, Asilomar, CA. *December 2012*
Generalized Lucas Bases
66. Young Mathematician's Conference, Columbus, OH. *July 2012*
Combinatorial Rearrangements on Arbitrary Graphs
67. Northwest Undergraduate Mathematics Symposium, Portland, OR. *March 2012*
Combinatorial Rearrangements on Arbitrary Graphs
68. WSU Graduate Seminar on Combinatorial Geometry, Pullman, WA. *(Quarterly) 2012-2013*
Various Topics

Posters

1. SIAM Workshop on Network Science, Boston, MA. *July 2016*
   Generalized Random Dot Product Models For Multigraphs
2. Dartmouth Graduate Student Poster Session, Hanover, NH. *April 2016*
   Generalized Dot Product Models for Weighted Networks
3. Dartmouth Graduate Student Poster Session, Hanover, NH. *April 2015*
   Multiplex Structures in the World Trade Web
4. WSU SURCA, Pullman, WA. *March 2013*
   Empirical Analysis of Space Filling Curves for Scientific Computing Applications
5. WSU SURCA, Pullman, WA. *April 2012*
   Combinatorial Rearrangements, Restricted Permutations, and Matrix Permanents

HONORS AND AWARDS

- Dartmouth Hannah Croasdale Award *2018*
  College-wide award for the graduating Ph.D. student that best exemplifies the qualities of a scholar.
- Dartmouth Graduate Student Teaching Award *2017*
  College-wide award for the graduate student who best exemplifies the qualities of a college educator.
- Dartmouth Graduate Fellowship *2014–18*
- NSF Graduate Research Fellowship: Honorable Mention *2014, 2015*
- Dartmouth GAANN Fellowship *2013*
- WSU Morris Knebelman Outstanding Senior Award *2013*
- WSU Department of Mathematics Outstanding Senior *2013*
- WSU Emeritus Society Award in the Physical Sciences *2013*
- WSU J. Russell and Mildred H. Vatnsdal Memorial Scholarship *2013*
- WSU SURCA Crimson Award: Computer Science and Mathematics *2012, 2013*
- WSU Auvil Undergraduate Scholars Fellowship *2012*
- WSU Leonard B. Kirschner Scholarship *2012*
- WSU College of Sciences Undergraduate Research Grant *2012*
- Norma C. Fuentes and Gary M Kirk Award for Excellence in Undergraduate Research *2012*

PROFESSIONAL SERVICE

Peer Reviewer

- Election Law Journal
- Transactions on Signal and Information Processing over Networks
- Multiscale Modeling and Simulation: A SIAM Interdisciplinary Journal
- International Conference on Learning Representations (ICLR)
- International Conference on Artificial Intelligence and Statistics (AISTATS)
- AAAI Conference on Artificial Intelligence (AAAI)
- International Conference on Machine Learning (ICML)
- ACM-SIAM Symposium on Discrete Algorithms (SODA)
- Neural Information Processing Systems (NeurIPS)
- Transactions on Pattern Analysis and Machine Intelligence (TPAMI)
- Chaos: An Interdisciplinary Journal of Nonlinear Science
- Involve: A Journal of Mathematics
- Entropy
- MATCH Communications in Mathematical and in Computer Chemistry

Appendix B

B Data and Materials

This appendix describes the data and materials that I relied on while performing this analysis and crafting this report.

B.i Data

The primary data sources and document repositories for the analysis in this report are publicly available, including the underlying geospatial data. I made use of data and documents from the following sources:

- Wisconsin-specific geospatial data and annotations (https://legis.wisconsin.gov/ltsb/gis/data/)
- Geospatial and population data from the US Census (https://www.census.gov/geographies/mapping-files/time-series/geo/tiger-line-file.html)
- Filings in this case (https://www.wicourts.gov/courts/supreme/origact/2021ap1450.htm)
  - Briefs, reports, maps, and expert materials submitted by the parties on December 15, 2021 and December 30, 2021 including material produced by parties pursuant to agreement on discovery
  - Supreme Court's November 30 order

B.ii Computational Libraries

The bulk of the computational work for this report was carried out using standard libraries of the Python programming language. I also used the following more specialized packages for specific computational tasks.

- [MAUP] github.com/mggg/maup
- [Gerrychain] github.com/mggg/gerrychain

Appendix C

C Metric Normalization for Figures 13 and 14

In order to make the metrics comparable, I normalized the scores for each proposed map to lie between an ideal value and the value in the 2011 enacted plan. More specifically, the population balance value is computed by assuming an ideal value of 0 person top-to-bottom deviation and computing the proportion of the enacted plan's deviation achieved by each proposal. For example, the CMS Assembly plan has exactly the same deviation as the 2011 plan (438 people), so it gets a score of 100%, while the Bewley plan has a deviation of 1104 people, so gets a score of $1104/438 \approx 252\%$. The county splits score is computed against an ideal value of splitting zero counties over the population requirement. For example, the CMS senate plan splits 27 counties over the population requirement so receives a score of $27/46 \approx 59\%$. Finally, for the compactness score, an ideal value corresponds to a Polsby-Popper score of 1, but as with Figures 9 and 10 I compute the complement by subtracting the score from 1 so as to obtain a metric where lower values correspond to better performing plans and I use the Polsby-Popper scores from the 2011 enacted plan to normalize the scores. For example, a circle would get a score of 1-1=0 and the CMS senate plan gets a score of (1-.26)/(1-.202).